Faculty Of Engineering & Architecture

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Lecture Title: Interpolation

Lecture No.:9

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Interpolation

- A function is said to interpolate a set of data points if passes through those points
- The function $y = p(x)$ interpolate the data points $(x_0, y_0), \cdots\cdots, (x_n, y_n)$ if $p(x_i) = y_i$ for each $0 \le i \le n$
- There is some polynomial $y = p(x)$ that runs through all the points :

1-The general polynomial interpolation

2-The Lagrange polynomial interpolation

3-The Newton divided differences polynomial interpolation

1- General method

Assume that n data points $(x_0, y_0), \ldots, (x_n, y_n)$ are given, and that we would like to find an interpolation polynomial.

The general polynomial can be writing as following

$$p_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \qquad (1)$$

Substituted n data points in the general form as following:

$$a_0 + a_1 x_0 + a_2 x_0^2 + \cdots + a_n x_0^n = y_0$$
$$a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n = y_1$$
$$(2)$$
$$a_0 + a_1 x_n + a_2 x_n^2 + \cdots + a_n x_n^n = y_n$$

Solve the equation (2) using the Cramer rule to find the unknown ($a_0, a_1, \ldots, a_n$) as following:

$$\Delta = \begin{vmatrix} 1 & x_0 & \cdots & x_0^n \\ 1 & x_1 & \cdots & x_1^n \\ \vdots & \vdots & & \vdots \\ 1 & x_n & \cdots & x_n^n \end{vmatrix}$$

$$\Delta_0 = \begin{vmatrix} y_0 & x_0 & \cdots & x_0^n \\ y_1 & x_1 & \cdots & x_1^n \\ \vdots & \vdots & & \vdots \\ y_n & x_n & \cdots & x_n^n \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & y_0 & \cdots & x_0^n \\ 1 & y_1 & \cdots & x_1^n \\ \vdots & \vdots & & \vdots \\ 1 & y_n & \cdots & x_n^n \end{vmatrix}$$

$$\Delta_n = \begin{vmatrix} 1 & x_0 & \cdots & y_0 \\ 1 & x_1 & \cdots & y_1 \\ \vdots & \vdots & & \vdots \\ 1 & x_n & \cdots & y_n \end{vmatrix}$$

$$\therefore a_0 = \frac{\Delta_0}{\Delta}, a_1 = \frac{\Delta_1}{\Delta}, \cdots, a_n = \frac{\Delta_n}{\Delta} \qquad (3)$$

Substitution equation (3) in equation (1) to find the interpolation polynomial passing through the data points

Example:

use the general interpolation to find a polynomial that passes through the points (0,1) , (1,2) ,(2,0) , (3,-1)

x	0	1	2	3
y	1	2	0	-1

Solution:

$$p_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad (1)$$

Substituted n data points in the general form as following:

$$a_0 + a_1 x_0 + a_2 x_0^2 + a_n x_0^3 = y_0$$

$$a_0 + a_1 x_1 + a_2 x_1^2 + a_n x_1^3 = y_1$$

$$a_0 + a_1 x_2 + a_2 x_2^2 + a_3 x_2^3 = y_2 \qquad (2)$$

$$a_0 + a_1 x_3 + a_2 x_3^2 + a_n x_3^3 = y_3$$

Solve the equation (2) using the Cramer rule to find the unknown ($a_0, a_1, \ldots, a_n$) as following:

$$\Delta = \begin{vmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{vmatrix} = 12$$

$$\Delta_0 = \begin{vmatrix} y_0 & x_0 & x_0^2 & x_0^3 \\ y_1 & x_1 & x_1^2 & x_1^3 \\ y_2 & x_2 & x_2^2 & x_2^3 \\ y_3 & x_3 & x_3^2 & x_3^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 2 & 4 & 8 \\ -1 & 3 & 9 & 27 \end{vmatrix} = 12$$

$$\Delta_1 = \begin{vmatrix} 1 & y_0 & x_0^2 & x_0^3 \\ 1 & y_1 & x_1^2 & x_1^3 \\ 1 & y_2 & x_2^2 & x_2^3 \\ 1 & y_3 & x_3^2 & x_3^3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 4 & 8 \\ 1 & -1 & 9 & 27 \end{vmatrix} = 46$$

$$\Delta_2 = \begin{vmatrix} 1 & x_0 & y_0 & x_0^3 \\ 1 & x_1 & y_1 & x_1^3 \\ 1 & x_2 & y_2 & x_2^3 \\ 1 & x_3 & y_3 & x_3^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 8 \\ 1 & 3 & -1 & 27 \end{vmatrix} = -42$$

$$\Delta_3 = \begin{vmatrix} 1 & x_0 & x_0^2 & y_0 \\ 1 & x_1 & x_1^2 & y_1 \\ 1 & x_2 & x_2^2 & y_2 \\ 1 & x_3 & x_3^2 & y_3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 0 \\ 1 & 3 & 9 & -1 \end{vmatrix} = 8$$

$$\therefore a_0 = \frac{\Delta_0}{\Delta} = \frac{12}{12} = 1,\ a_1 = \frac{\Delta_1}{\Delta} = \frac{46}{12} = 3.833,\ a_2 = \frac{\Delta_2}{\Delta} = \frac{-42}{12} = -3.5,\ a_3 = \frac{\Delta_3}{\Delta} = \frac{8}{12} = 0.667 \qquad (3)$$

Substitution equation (3) in equation (1) to find the interpolation polynomial passing through the data points

$$p_3(x) = 1 + 3.833\,x - 3.5\,x^2 + 0.667\,x^3$$