Faculty Of Engineering & Architecture

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# 1- Newton method

- Newton method also called the Newton – Raphson method
- The Newton method is faster converges than the other method
- To find the root of $f(x)=0$, a starting guess $x_0$ is given
- The formula :

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$ for I = 0 ,1 ,2,……

## Example

Find the Newton method formula for the equation $x^3 + x - 1 = 0 \quad x_0 = -0.7$

Since $f'(x) = 3x^2 + 1$, the formula is given by

$$x_{i+1} = x_i - \frac{x_i^3 + x_i - 1}{3x_i^2 + 1}$$

$$= \frac{2x_i^3 + 1}{3x_i^2 + 1}$$

Iterating this formula from initial guess $x_0 = -0.7$ yields

$$x_1 = \frac{2x_0^3 + 1}{3x_0^2 + 1} = \frac{2(-0.7)^3 + 1}{3(-0.7)^2 + 1} \approx 0.1271$$

$$x_2 = \frac{2x_1^3 + 1}{3x_1^2 + 1} = \frac{2(0.1271)^3 + 1}{3(0.1271)^2 + 1} \approx 0.9577$$

These steps are given in the following table:

| i | $x_i$ | $e_i = \lvert x_i - r \rvert$ | $e_i / e_{i-1}^2$ |
|---|---|---|---|
| 0 | -0.70000000 | 1.38232780 | |
| 1 | 0.12712551 | 0.55520300 | 0.2906 |
| 2 | 0.95767812 | 0.27535032 | 0.8933 |
| 3 | 0.73482779 | 0.05249999 | 0.6924 |
| 4 | 0.6845917 | 0.002263 | 0.8214 |

| | | | |
|---|---|---|---|
| | 7 | 97 | |
| 5 | 0.68233217 | 0.00000437 | 0.8527 |
| 6 | 0.68232780 | 0.00000000 | 0.8541 |

# Newton – Raphson algorithm

To find a solution to $f(x)=0$ given an initial approximation $x_0$

Input: initial approximation $x_0$ : tolerance (tol): maximum number of iterations $N_0$

Output: approximate solution (x) or message of failure

Step 1 : set i=1

Step 2 : while I $\le N_0$ do steps 3-6

Step 3: set $x = x_0 - f(x_0)/f'(x_0)$ (compute $x_0$)

Step 4 : if $|x - x_0| < tol$ then

Output (x) ; (procedure completed successfully)

Stop .

Step5 : set i=i+1

Step6 : set $x_0 = x$ (update $x_0$)

Step7 : output (method failed after $N_0$ iteration

Stop

# Exercise

1-Show that $f(x) = x^3 - x - 1$ has exactly one zero in the interval [1,2]. Approximate the zero to within $10^{-2}$ using the bisection method

2-Use the bisection method to find solutions accurate to within $10^{-2}$ for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on

a)[-2,-1] b) [0,2] c) [2,3] d) [-1,0]

## False position method:

Is similar to the bisection method, (given an interval [a,b] that bracket a root, assume that $f(a)\,f(b) < 0)$ , but where the midpoint is replaced

$$c = \frac{bf(a) - af(b)}{f(a) - f(b)}$$

The new interval, either [a,c] or [c,b] , is chosen according to whether $f(a)\,f(c) < 0$ or $f(c)\,f(b) < 0$, respectively, and still brackets a root.

## Method of false position algorithm:

Given interval [a,b] such that $f(a)\,f(b) < 0$

For $i = 1., 2, 3, \cdots$

$$c = \frac{bf(a) - af(b)}{f(a) - f(b)}$$

If $f(c) = 0$, stop, end

If $f(a)\,f(c) < 0$

$b = c$

Else

$a = c$

End

End

The method of false position at first appears to be an improvement on both

# the bisection method and the secant method

## Example:

Apply the method of false position on initial interval [0,1] to find the root $r=0$ of

$$f(x)=x^3-4x+1$$

## Solution:

Given a=0 ,b=1

$$f(x)=x^3-4x+1$$

$$f(0)=0^3-4(0)+1=1$$

$$f(1)=(1)^3-4(1)+1=-2$$

$$f(0)\,f(1)<0$$

The new point

$$c = \frac{bf(a) - af(b)}{f(a) - f(b)} = \frac{(0)(-2) - (1)(1)}{-2-1} = \frac{1}{3}$$

$$f(c) = f(1/3) = (1/3)^3 - 4(1/3) + 1 = -0.2963$$

| n | $a_n$ | $b_n$ | $c_n$ | $f(c_n)$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 0.333333 | -0.296296 |
| 2 | 0 | 0.333333 | 0.2571400 | -0.00038552 |
| 3 | 0 | 0.257140 | 0.2542000 | -0.000012546 |
| 4 | 0 | 0.254200 | 0.2541018 | -0.00000041 |

| | | | | |
|---|---|---|---|---|
| | | | | 2 |
| 5 | 0 | 0.254100 | 0.2541017 | |

## Secant method:

The secant method is similar to the Newton's method, but replaces the derivative by a difference quotient. Geometrically, the tangent line is replaced with a line through the two last known guesses. The intersection point of the "secant line" is the new guess.

An approximation for the derivative at the current guess $x_i$ is the difference quotient

$$\frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

A straight replacement of this approximation for $f'(x_i)$ in Newton's method yields the secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \quad for \quad i = 1,2,3\cdots$$

$x_0, x_1$ = Initial guesses

Unlike fixed – point iteration and Newton's method, two starting guesses are needed to begin the secant method.

# Example:

Apply the secant method with starting guesses $x_0 = 0,\ x_1 = 1$ to find the root of

$f(x) = x^3 + x - 1$

## Solution:

The formula gives

$$x_{i+1} = x_i - \frac{(x_i^3 + x_i - 1)(x_i - x_{i-1})}{x_i^3 + x_i - (x_{i-1}^3 + x_{i-1})}$$

Starting with $x_0 = 0$ and $x_1 = 1$ we compute

$$x_2 = 1 - \frac{(1)(1-0)}{1+1-0} = \frac{1}{2}$$

$$x_3 = \frac{1}{2} - \frac{-\frac{3}{8}(1/2 - 1)}{-\frac{3}{8} - 1} = \frac{7}{11}$$

# Further iterates form the following table

| $i$ | $x_i$ |
|---|---|
| 0 | 0.00000000000000 |
| 1 | 1.00000000000000 |
| 2 | 0.50000000000000 |
| 3 | 0.63636363636364 |
| 4 | 0.69005235602094 |

| | |
|---|---|
| 5 | 0.68202041964819 |
| 6 | 0.68232578140989 |
| 7 | 0.68232780435903 |
| 8 | 0.68232780382802 |
| 9 | 0.68232780382802 |