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Lecture Title: Non – linear algebraic equation

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Non – linear algebraic equation

In this will describe several different methods for solving $f(x)=0$

Methods:

- Interval halving (bisection)
- Newton method
- False position method
- Secant method
- Fixed – point : $x=g(x)$ method

1-Interval halving:

- Its strategy is to begin with two values of x=a and b that bracket a root of $f(x)=0$

- It determines that the values x=a and x=b do bracket a root by finding that

$$f(a) * f(b) < 0$$

Because they are of opposite signs

- The method then successively divides the interval in half and replaces one end point with the midpoint so that again the root is bracket

$$c = \frac{a+b}{2}$$

- Error after (n) iterations < $\left|\frac{b-a}{2^{n+1}}\right|$

- A solution is correct within (p)

decimal places if the error is less than 0.5×10^{-p}

Example

Find a root of the function $f(x)=x^3+x-1$ by using the bisection method on the interval [0,1]

Solution

As noted, $f(a_0)*\ f(b_0)=(-1)(1)<0$, so a root exists in the interval.

The interval midpoint is $c_0=1/2$.

The first step consists of evaluating $f(1/2)=-3/8<0$

Choosing the new interval $[a_1,b_1]=[1/2,1]$

since $f(1/2)\,f(1) =< 0$

The second step consists of evaluating

$f(c_1) = f(3/4) = 11/64 > 0$

Leading to the new interval

$[a_2, b_2] = [1/2, 3/4]$

Continuing in this way yields the following interval

i	a_i	$f(a_i)$	c_i	$f(c_i)$	b_i	$f(b_i)$
0	0.0000	-	0.5000	-	1.0000	+
1	0.5000	-	0.7500	+	1.0000	+

2	0.5000	-	0.6250	-	0.7500	+
3	0.6250	-	0.6875	+	0.7500	+
4	0.6250	-	0.6562	-	0.6875	+
5	0.6562	-	0.6719	-	0.6875	+
6	0.6719	-	0.6797	-	0.6875	+
7	0.6797	-	0.6836	+	0.6875	+
8	0.6797	-	0.6816	-	0.6836	+

9	0.6816	-	0.6826	+	0.6836	+

$c_{10} \approx 0.6821$

The interval contain a root is

[0.6816, 0.6826]

The root is 0.6821 $\pm$ 0.0005

Example

Using the bisection method to find a root of $f(x) = \cos x - x$ in the interval [0,1] to within six correct places

Solution

$$\frac{(b-a)}{2^{n+1}} < 0.5 \times 10^{-p}$$

$$\frac{1}{2^{n+1}} < 0.5 \times 10^{-6}$$

$$n > \frac{6}{\log 2} \approx \frac{6}{0.301} \approx 19.9$$

An algorithm for halving the interval (bisection):

To determine a root of $f(x) = 0$ that is accurate within a specified tolerance value given initial interval [a,b] such that

$$f(a)\, f(b) < 0$$

Repeat

Set (c) = (a+b)/2

If f(c)f(a)<0

Set b=c

Else set a=c

End if

Until ($|a-b|<$tolerance value) or f(c) =0

The final value of c approximates the root it is error not more than (1/2)$|a-b|$.

Note: the method may give a false root if (f(x) is discontinuous on [a, b]).