Faculty Of Engineering & Architecture

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Course Title: Numerical methods Course code: MAT221

Lecture Title: Itrative method of solving linear system of equation (Gauss – Seidel method)

Lecture No.:6

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Itrative method of solving linear system of equation

Gauss – Seidel method

Using equation

Example

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Rewrite the equations, solving for the unknown

$$x_1 = \frac{1}{a_{11}}[b_1 - a_{12}x_2 - a_{13}x_3]$$

$$x_2 = \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3]$$

$$x_3 = \frac{1}{a_{33}}[b_3 - a_{31}x_1 - a_{32}x_2]$$

Then, iterate as in fixed point iteration, starting with an initial guess

$$x_1^{(k+1)} = \frac{1}{a_{11}}[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}]$$

$$x_2^{(k+1)} = \frac{1}{a_{22}}[b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}]$$

$$x_3^{(k+1)} = \frac{1}{a_{33}}[b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}]$$

Stop until the condition

$$\left|x_i^{(k+1)} - x_i^{(k)}\right| \le \varepsilon$$

Where: ε = given tolerance

Example:

Apply the Gauss – Seidel method to the system 3u+v=5, u+2v=5

Solution:

Begin by solving the first equation for u and the second equation for v. we will use the initial guess $(u_0, v_0) = (0, 0)$. We have

$$u = \frac{5-v}{3} \quad , \quad v = \frac{5-u}{2}$$

The two equations are iterated

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \dfrac{5-v_0}{3} \\ \dfrac{5-u_1}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{5-0}{3} \\ \dfrac{5-5/3}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{5}{3} \\ \dfrac{5}{3} \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \dfrac{5-v_1}{3} \\ \dfrac{5-u_2}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{5-5/3}{3} \\ \dfrac{5-10/9}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{10}{9} \\ \dfrac{35}{18} \end{bmatrix}$$

$$\begin{bmatrix} u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} \dfrac{5-v_2}{3} \\ \dfrac{5-u_3}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{5-35/18}{3} \\ \dfrac{5-55/54}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{55}{54} \\ \dfrac{215}{108} \end{bmatrix}$$

Note the difference between Gauss – seidel and Jacobi: the definition of v_1 uses u_1, not u_0. We see the approach to the solution [1,2] as with Jacobi method, but somewhat more accurately at same number of steps. Gauss – seidel often converges faster than Jacobi if the method is convergent.