Faculty Of Engineering & Architecture

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Lecture Title: Gauss elimination method

Lecture No.:4

Course coordinator: Dr. Ibrahim M.Ibrahim

Contact: 00249917066160 – ataalmanan@yahoo.com

Gauss elimination method

The general description of gauss method:

The method consists of two stages:

- The elimination stage
- The back substitution stage

The elimination stage:

In this stage the augmented form of the system is transformed into an upper triangular form by sequence of elementary rows operation carried out in (n-1) steps

Thus

Starting with

$$[A^{(1)}|b^{(1)}] = [A|b]$$

The kth step transforms

$[A^{(k)}|b^{(k)}]$ into $[A^{(k+1)}|b^{(k+1)}]$

Thus $[A^{(n)}|b^{(n)}]$ is the final (upper triangular)

Example

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$[A^{(1)}|b^{(1)}] = \left[\begin{array}{ccc|c} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & b_1^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & b_3^{(1)} \end{array}\right]$$

Using elementary rows operation:

$$E_2 - m_{21} E_1 \rightarrow E_2 \qquad m_{21} = a_{21} / a_{11}$$

$$E_3 - m_{31} E_1 \rightarrow E_3 \qquad m_{31} = a_{31} / a_{11}$$

$$[A^{(2)} | b^{(2)}] = \left[\begin{array}{ccc|c} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & b_1^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & b_2^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & b_3^{(2)} \end{array}\right]$$

$$E_3 - m_{32} E_2 \rightarrow E_3 \qquad m_{32} = a_{32} / a_{22}$$

$$[A^{(3)} | b^{(3)}] = \left[\begin{array}{ccc|c} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & b_1^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & b_2^{(2)} \\ 0 & 0 & a_{33}^{(3)} & b_3^{(3)} \end{array}\right]$$

Since the new linear system is triangular

$$a_{11}^{(1)} x_1 + a_{12}^{(1)} x_2 + a_{13}^{(1)} x_3 = b_1^{(1)}$$

$$a_{22}^{(2)} x_2 + a_{23}^{(2)} x_3 = b_2^{(2)}$$

$$a_{33}^{(3)} x_3 = b_3^{(3)}$$

The back substitution stage

This is called the solution stage it solve the triangular system

$$x_3 = b_3^{(3)} / a_{33}^{(3)}$$

$$x_2 = (b_2^{(2)} - a_{23}^{(2)} x_3) / a_{22}^{(2)}$$

$$x_1 = (b_1^{(1)} - a_{12}^{(1)} x_2 - a_{13}^{(1)} x_3) / a_{11}^{(1)}$$

Example

Solve the following system using gauss elimination method

$$x_1 + 2x_2 + 3x_3 = 5$$

$$2x_1 + 5x_2 + 3x_3 = 3$$

$$x_1 + \quad + 8x_3 = 17$$

$$[A^{(1)}|b^{(1)}] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 5 & 3 & 3 \\ 1 & 0 & 8 & 17 \end{array}\right]$$

$$E_2 - m_{21}E_1 \rightarrow E_2 \qquad m_{21} = a_{21}/a_{11} = 2/1 = 2$$

$$E_2 - 2E_1 \rightarrow E_2$$

$$E_3 - m_{31}E_1 \rightarrow E_3 \qquad m_{31} = a_{31}/a_{11} = 1/1 = 1$$

$$E_3 - E_1 \rightarrow E_3$$

$$[A^{(2)}|b^{(2)}] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -3 & -7 \\ 0 & -2 & 5 & 12 \end{array}\right]$$

$$E_3 - m_{32}E_2 \rightarrow E_3 \qquad m_{32} = a_{32}/a_{22} = -2/1 = -2$$

$$E_3 - (-2)E_2 \rightarrow E_3$$

$$[A^{(3)}|b^{(3)}] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & -1 & -2 \end{array}\right]$$

Since the new linear system is triangular

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ x_2 - 3x_3 &= -7 \\ -x_3 &= -2 \end{aligned}$$

The back substitution

$$x_3 = b_3^{(3)} / a_{33}^{(3)} = -2/-1 = 2$$

$$x_2 = (b_2^{(2)} - a_{23}^{(2)} x_3)/a_{22}^{(2)} = (-7 + 3 * 2)/1 = -1$$

$$x_1 = (b_1^{(1)} - a_{12}^{(1)} x_2 - a_{13}^{(1)} x_3)/a_{11}^{(1)} = (5 - 2(-1) - 3 * 2)/1 = 1$$

The gauss elimination algorithm

Input

* read(n)

* for i=1 to n do

* For j=1 to n+1 do

* read(aij)

Elimination

- For k=1 to n-1 do
- For i=k+1 to n do
- $m = a_{ik} / a_{kk}$
- For j=k+1 to n+1 do
- a_{ij} =aij-m*akj

Back substitution

* $x_n = a_{n,n+1} / a_{nn}$

* for i= n-1 to 1 do

* x(i)=a(i,n+1)

* for j =i+1 to n do

* x(i)=x(i)-a(ij)*x(j)

- * $x(i) = x(i)/a(ii)$

Out put

- For i=1to n do
- Write (xi)

Exercise

Solve the following systems using

1-Inverse of coefficient matrix

2-Gauss elimination

$$2x_1 - 3x_2 + x_3 = 2$$
$$x_1 + x_2 - x_3 = -1$$
$$-x_1 + x_2 - 3x_3 = 0$$

$$2x_1 - 3x_2 + x_3 = 6$$
$$x_1 + x_2 - x_3 = 4$$
$$-x_1 + x_2 - 3x_3 = 5$$

$$2x_1 - 3x_2 + x_3 = 0$$
$$x_1 + x_2 - x_3 = 1$$
$$-x_1 + x_2 - 3x_3 = -3$$

$$2x_1 - 3x_2 + x_3 = -1$$
$$x_1 + x_2 - x_3 = 0$$
$$-x_1 + x_2 - 3x_3 = 0$$