Faculty Of Engineering & Architecture

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Lecture Title: Solution of linear systems of equation

Lecture No.:3

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Solution of linear systems of equation

The problem is to find $x \in R^n$, which satisfies $Ax = b$

Where A: is a known mxn matrix , b: is a known m-vector and x: the vector of unknowns

$$A_{m\times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad b_{m\times 1} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \quad x_{n\times 1} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

The system $Ax = b$ is

(i) Well-determined if m=n (square system)

(ii) Under - determined if $m < n$

(iii) Over – determined if $m > n$

Well- determined system (m=n)

Solving linear systems by matrix inversion

If A is an invertible nxn matrix, then for each nx1 matrix b, the system of equation $Ax = b$ has exactly one solution, namely $x = A^{-1}b$

Example

Solve the following systems using the inverse of the matrix

$$(i) \quad \begin{aligned} x_1 + x_2 &= 0 \\ 2x_1 + x_2 &= 1 \end{aligned}$$

Solution:

In matrix form this system can be written as $Ax = b$, where

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad , \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad , \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Use any method to find A^{-1}

$$A^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$$

The solution of the system is

$$x = A^{-1}b = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Or $x_1 = 1 \, . \, x_2 = -1$

$$(ii) \quad x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 + \quad + 8x_3 = 17$$

Solution:

In matrix form this system can be written as $Ax = b$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

Use any method to find A^{-1}

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

The solution of the system is

$$x = A^{-1}b = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Or $x_1 = 1 .\ x_2 = -1 ,\ x_3 = 2$

$(iii)\quad x_1 + 2x_2 - x = 4$

$2x_1 - 3x_2 + x_3 = -1$

$5x_1 + 7x_2 + 2x_3 = -1$

Solution:

In matrix form this system can be written as

$$Ax = b$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -3 & 1 \\ 5 & 7 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$$

Use any method to find A^{-1}

$$A^{-1} = \frac{-1}{40} \begin{bmatrix} -13 & -11 & -1 \\ 1 & 7 & -3 \\ 29 & 3 & -7 \end{bmatrix}$$

The solution of the system is

$$x = A^{-1} b$$

$$x = \frac{-1}{40}\begin{bmatrix} -13 & -11 & -1 \\ 1 & 7 & -3 \\ 29 & 3 & -7 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$$

$$u = (1\ ,\ 0\ ,\ -3)$$