



Faculty Of Engineering & Architecture

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Course Title: Numerical methods **Course code:**
MAT221

Lecture Title:
Ordinary differential
equation

Lecture No.:13

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Ordinary differential equation

Initial value problem: find $y(x)$ if

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \\ x \in [a, b] \end{cases}$$

1-Euler method:

$$y_{n+1} = y_n + h f(x_n, y_n)$$

Where

$$h = \frac{b - a}{n}$$

Example

Find $y(1)$ if

$$y' = y - x \quad , \quad y(0) = 2 \quad , \quad h = 0.25$$

$$x_0 = 0 \quad , \quad y_0 = 2$$

$$x_1 = x_0 + h = 0.25, \quad x_2 = x_1 + h = 0.5, \quad x_3 = x_2 + h = 0.75, \quad x_4 = x_3 + h = 1$$

$$y_{n+1} = y_n + h f(x_n, y_n) \quad , \quad n = 0, 1, 2, 3$$

$$y_1 = y_0 + h f(x_0, y_0) = 2 + 0.25(2 - 0) = 2.5$$

$$y_2 = y_1 + h f(x_1, y_1) = 2.5 + 0.25(2.5 - 0.25) = 3.0625$$

$$y_3 = y_2 + h f(x_2, y_2) = 3.0625 + 0.25(3.0625 - 0.5) = 3.703125$$

$$y_4 = y_3 + h f(x_3, y_3) = 3.703125 + 0.25(3.703125 - 0.75) = 4.441$$

$$y(1) = y_4 = 4.441$$

The true solution is

$$y(x) = e^x + x + 1 \Rightarrow y(1) = e + 1 + 1 = 4.7182$$

Example



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Find

$y(1)$

if

$$y' = y - x \quad , \quad y(0) = 2 \quad , \quad h = 0.1, 0.05, 0.01, 0.005$$

x_n	y_n				True solution
	H=0.1	H=0.05	H=0.01	H=0.005	
0.0	2.0000	2.0000	2.0000	2.0000	2.0000
0.1	2.2000	2.2025	2.2046	2.2049	2.2052
0.2	2.4100	2.4155	2.4202	2.4208	2.4214



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0.3	2.631 0	2.610 1	2.647 8	2.6489	2.6499
0.4	2.864 1	2.877 5	2.888 9	2.8903	2.8918
0.5	3.110 5	3.128 9	3.144 6	3.1467	3.1487
0.6	3.371 6	3.395 9	3.416 7	3.4194	3.4221
0.7	3.648 7	3.679 9	3.706 8	3.7102	3.7138
0.8	3.943 6	3.982 9	4.016 7	4.0211	4.0255
0.9	4.257 9	4.306 6	4.348 6	4.3541	4.3596



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1.0	4.593 7	4.653 3	4.704 8	4.7115	4.7183
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2-Runge-kutta method of order four (RK4)

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = f(x_n + h, y_n + k_3)$$

Example

Find $y(1)$ if
 $y' = y - x$, $y(0) = 2$, $h = 0.1$ using
Runge-kutta method

x_n	$h = 0.1$	True solution
0.0	2.000000 0	2.000000 0
0.1	2.205170 8	2.205170 9
0.2	2.421402 6	2.421402 8
0.3	2.649858 5	2.649858 8



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0.4	2.891824 2	2.891824 7
0.5	3.148720 6	3.148721 3
0.6	3.422118 0	3.422118 8
0.7	3.713751 6	3.713752 7
0.8	4.025539 6	4.025540 9
0.9	4.359601 4	4.359603 1
1.0	4.718279 7	4.718281 8

Exercises

1-Using initial condition $y(0) = 1$ and step $h = 1/4$, calculate $y(1)$ use Euler method and comparing with the correct solution for the following equations

$$(i) y' = x \quad (ii) y' = x^2 y \quad (iii) y' = 2(x + 1) y$$

2-Using initial condition $y(0) = 0$ and step $h = 1/4$, calculate $y(1)$ use Euler method and comparing with the correct solution for the following equations



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$$(i) y' = x + y \quad (ii) y' = x - y \quad (iii) y' = 4x - 2y$$

3-Repeat the solution of (1) and (2) using Runge-kutta method



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