Faculty Of Engineering & Architecture

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Course Title: Numerical methods Course code: MAT221

Lecture Title: Numerical integration

Lecture No.:12

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Numerical integration

Newton-cotes formulas for numerical integration

1-Trapezoid rule

$$\int_a^b f(x)\,dx = \frac{h}{2}(f_a + f_b)$$

$$h = b - a$$

2-Simpson rule

$$\int_a^b f(x)\,dx = \frac{h}{3}(f_0 + 4f_1 + f_2)$$

$$h = \frac{b-a}{2}$$

$$f_0 \equiv f_a, \quad f_1 = f(a_a + h), \quad f_2 \equiv f_b$$

Example

Apply the trapezoid rule and Simpson rule to approximate

$$\int_1^2 \ln x \, dx$$

The trapezoid rule estimates that

$$\int_1^2 \ln x \, dx \approx \frac{h}{2}(f_a + f_b) = \frac{1}{2}(\ln 1 + \ln 2) = \frac{\ln 2}{2} \approx 0.3466$$

The integral can be computed exactly by using integration by parts:

$$\int_1^2 \ln x \, dx = x \ln x \Big|_1^2 - \int_1^2 dx$$

$$= 2 \ln 2 - 1 \ln 1 - 1 \approx 0.386294$$

Simpson rule yields the estimate

$$\int_1^2 \ln x \, dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2) = \frac{0.5}{3}\left(\ln 1 + 4\ln\frac{3}{2} + \ln 2\right) \approx 0.3858$$

Composite Newton-cotes formulas

1-Composite trapezoid rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left(f_0 + 2\sum_{i=1}^{n-1} f_i + f_n\right)$$

$$h = \frac{b-a}{n} \quad , \; f_0 \equiv f_a \, , \; f_1 = f_a + ih \, , \; f_n \equiv f_b$$

N= the number of divisions

2-Composite Simpson rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left(f_0 + 4\sum_{1}^{m-1} f_{(odd)} + 2\sum_{2}^{m-2} f_{(even)} + f_m \right)$$

$$h = \frac{b-a}{2m} \quad , \; f_0 \equiv f_a \; , \; f_1 = f_a + ih \; , \; f_m \equiv f_b$$

M=the number of divisions & must be even number

Example

Carry out four- panel approximation of $\int_1^2 \ln x\,dx$ using the composite trapezoid rule and composite Simpson rule

For the composite trapezoid rule on [1,2] four panel means that h=1/4. The approximation

$$\int_1^2 \ln x\,dx = \frac{h}{2}\left(f_0 + 2\sum_{i=1}^{n-1} f_i + f_n\right)$$

$$h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$= \frac{1}{8}[\ln 1 + \ln 2 + 2(\ln 5/4 + \ln 6/4 + \ln 7/4)]$$

$$\approx 0.3837$$

A four-panel simpson rule sets h=1/8. The approximation is

$$\int_1^2 \ln x\, dx = \frac{1/8}{3}\left(f_0 + 4\sum_1^4 f_{(odd)} + 2\sum_2^3 f_{(even)} + f_8 \right)$$

$$= \frac{1}{24}[\ln 1 + \ln 2 + 4(\ln 9/8 + \ln 11/8 + \ln 13/8 + \ln 15/8)$$

$$+ 2(\ln 5/4 + \ln 6/4 + \ln 7/4)]$$

$$\approx 0.386292$$

Exercises

1-Apply the composite trapezoid rule with n=1,2 and 4 panels to approximate the integral, compute the error by comparing with the correct value

$$(i)\int_0^1 x^2\, dx \qquad (ii)\int_0^{\pi/2} \cos x\, dx \qquad (iii)\int_0^1 e^x\, dx$$

2-Apply the composite Simpson rule

with n=1,2 and 4 panel to the integral in exercise 1 and report the errors

3-Apply the composite Simpson rule with n=1,2 and 4 panel to the integral and report the errors

$$(i)\int_0^1 \frac{dx}{1+x^2} \qquad (ii)\int_0^{\pi} x\cos x\,dx \qquad (iii)\int_0^1 xe^{x}\,dx$$