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1-Lagrange interpolation method

Assume that n data points $(x_0, y_0), \ldots, (x_n, y_n)$ are given, and those we would like to find an interpolation polynomial passes through these points

The Lagrange interpolation polynomial can be writing as following:

$$p_n(x) = L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2 + \cdots + L_n(x)y_n$$

Where:

$$L_0(x) = \frac{(x - x_1)(x - x_2)\ldots\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots\ldots(x_0 - x_n)}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)\ldots\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots\ldots(x_1 - x_n)}$$

$$L_n(x) = \frac{(x - x_0)(x - x_1)\ldots\ldots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)\ldots\ldots(x_n - x_{n-1})}$$

Example

Use Lagrange interpolation to find a polynomial that passes through the points (0,-3) , (1,1) , (2,2) , (4,7)

Solution:

The Lagrange interpolation polynomial can be writing as following:

$$p_3(x) = L_0(x)\,y_0 + L_1(x)\,y_1 + L_2(x)\,y_2 + L_3(x)\,y_3$$

Where:

$$L_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$= \frac{(x-1)(x-2)(x-4)}{(-1)(-2)(-4)}$$

$$= \frac{(x-1)(x-2)(x-4)}{-8}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)}$$

$$= \frac{x(x-2)(x-4)}{1(-1)(-3)}$$

$$= \frac{x(x-2)(x-4)}{3}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$= \frac{x(x-1)(x-4)}{(2)(1)(-2)}$$

$$= \frac{x(x-1)(x-4)}{-4}$$

$$L_3(x) = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

$$= \frac{x(x-1)(x-2)}{(4)(3)(2)}$$

$$= \frac{x(x-1)(x-2)}{24}$$

$$p_3(x) = \frac{3}{8}(x-1)(x-2)(x-4) + \frac{1}{3}x(x-2)(x-4) - \frac{1}{2}x(x-1)(x-4) + \frac{7}{24}x(x-1)(x-2)$$

Newton divided differences

- Newton divided differences give a particularly simple way to write the interpolating polynomial.
- Given n data points , the result will be a polynomial of degree at most n-1
- The idea of divided differences is fairly simple, but some notation needs to be mastered first
- Define the divided differences, which are the real numbers

$$f[x_k] = f(x_k)$$

$$f[x_k, x_{k+1}] = \frac{f[x_{k+1}] - f[x_k]}{x_{k+1} - x_k}$$

$$f[x_k, x_{k+1}, x_{k+2}] = \frac{f[x_{k+1}, x_{k+2}] - f[x_k, x_{k+1}]}{x_{k+2} - x_k}$$

$$f[x_k, x_{k+1}, x_{k+2}, x_{k+3}] = \frac{f[x_{k+1}, x_{k+2}, x_{k+3}] - f[x_k, x_{k+1}, x_{k+2}]}{x_{k+3} - x_k}$$

$$f[x_k, x_{k+1}, \ldots, x_{k+m}] = \frac{f[x_{k+1}, x_{k+2}, \ldots, x_{k+m}] - f[x_k, x_{k+1}, \ldots, x_{k+m-1}]}{x_{k+m} - x_k}$$

- View the data points given by some function f , and list the data points in a table:

x_i	F[ ]	F[,]	F[, ,]	
x_0	$f[x_0]$			
		$f[x_0,x_1]=\frac{f[x_1]-f[x_0]}{x_1-x_0}$		
x_1	$f[x_1]$		$f[x_0,x_1,x_2]=\frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$	
		$f[x_1,x_2]=\frac{f[x_2]-f[x_1]}{x_2-x_1}$		$f[x_0,x_1,x_2,x_3]=\frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$
x_2	$f[x_2]$		$f[x_1,x_2,x_3]=\frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$	
		$f[x_2,x_3]=\frac{f[x_3]-f[x_2]}{x_3-x_2}$		
x_3	$f[x_3]$			

- These numbers are the

coefficients of the interpolating polynomial for the data points, which is given by the Newton divided differences formula:

$$p_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots\cdots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Example:

Use divided differences to find the interpolating polynomial passing through the points (0,1) , (2,2) , (3,4)

Solution:

Applying the definitions of divided differences leads to the following table:

x	F[ ]	F[,]	F[, ,]
0	1		
		0.5	
2	2		0.5
		2	
3	4		

This table is computed using the definition of the divided differences

The interpolating polynomial can be

written as:

$$p_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$= 1 + 0.5(x - 0) + 0.5(x - 0)(x - 2) = 0.5x^2 - 0.5x + 1$$

$$p_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$= 1 + 0.5(x - 0) + 0.5(x - 0)(x - 2) = 0.5x^2 - 0.5x + 1$$