# National University

# Int. to Electrical Eng.

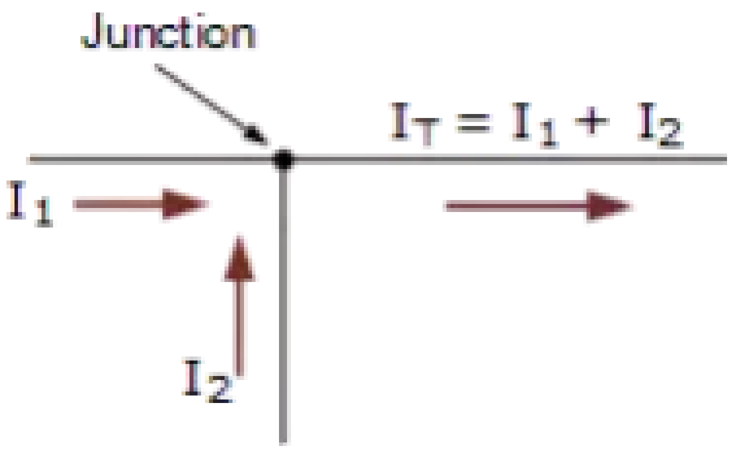
# Kirchhoff's Laws

# Introduction:

- **Kirchhoff's laws** are two laws that deal with the current and voltage in the model of electrical circuits.
- They were first described in 1845 by German physicist Gustav Kirchhoff.
- This generalized the work of Georg Ohm.
- Widely used in electrical engineering.

## 1. Kirchhoff’s Current Law

is Kirchhoff's first law that deals with the conservation of charge entering and leaving a junction, and is one of the fundamental laws used for circuit analysis.



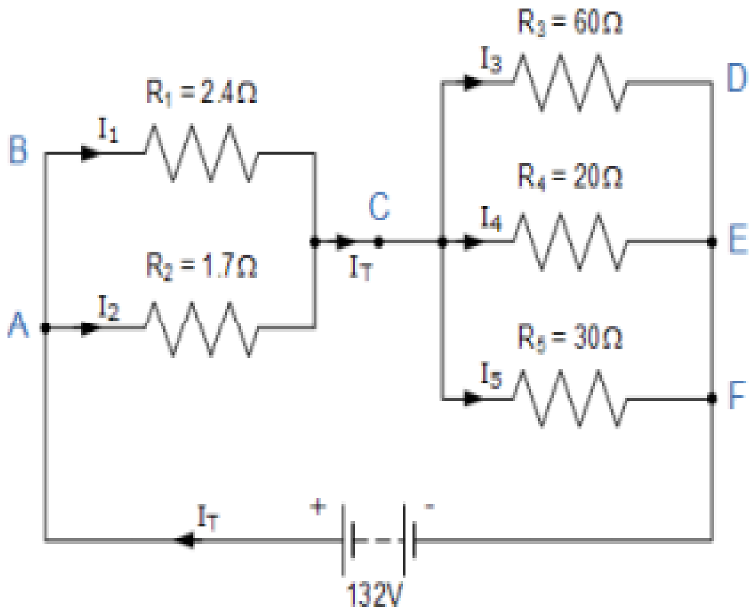

This law states that, for any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node;

**Σ I in = Σ I out .**

$I_T = I_1 + I_2$ .

$I_T - (I_1 + I_2) = 0$.

# Example No1:




first calculate the circuits total current, I . Ohms law tells us that I = V/R and as we know the value of V, 132 volts, we need to calculate the circuit resistances as follows:

**Circuit Resistance $R_{AC}$**

$$\frac{1}{R_{(AC)}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2.4} + \frac{1}{1.7}$$

$$\frac{1}{R_{(AC)}} = \frac{1}{1} \qquad \therefore R_{(AC)} = 1\Omega$$

**Circuit Resistance $R_{CF}$**

$$\frac{1}{R_{(CF)}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30}$$

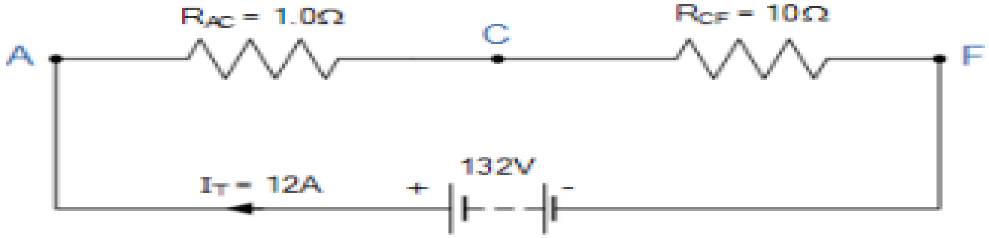
$$\frac{1}{R_{(CF)}} = \frac{1}{0.1} \qquad \therefore R_{(CF)} = 10\Omega$$

# Continued:

$$R_T = R_{(AC)} + R_{(CF)} = 1 + 10 = 11\Omega$$

$$I_T = \frac{V}{R_T} = \frac{132}{11} = 12\,\text{Amps}$$

**Kirchhoff's Current Law Equivalent Circuit**




Therefore, V = 132V, $R_{AC}$ = 1Ω, $R_{CF}$ = 10Ω's and $I_T$ = 12A.

# Continued:

$$V_{AC} = I_T \times R_{AC} = 12 \times 1 = 12\ \text{Volts}$$

$$V_{CF} = I_T \times R_{CF} = 12 \times 10 = 120\ \text{Volts}$$

$$I_1 = \frac{V_{AC}}{R_1} = \frac{12}{2.4} = 5\ \text{Amps}$$

$$I_2 = \frac{V_{AC}}{R_2} = \frac{12}{1.7} = 7\ \text{Amps}$$

$$I_3 = \frac{V_{CF}}{R_3} = \frac{120}{60} = 2\ \text{Amps}$$

$$I_4 = \frac{V_{CF}}{R_4} = \frac{120}{20} = 6\ \text{Amps}$$

$$I_5 = \frac{V_{CF}}{R_5} = \frac{120}{30} = 4\ \text{Amps}$$

# Continued:

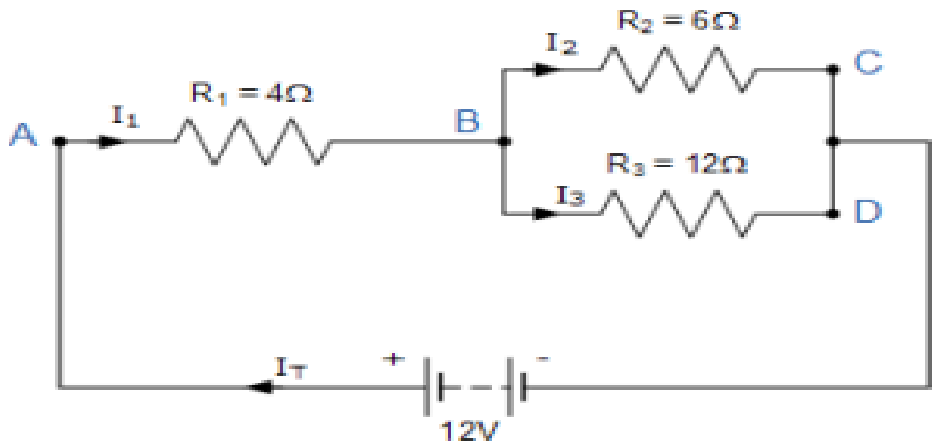
At node C $\sum I_{IN} = \sum I_{OUT}$

$$I_T = I_1 + I_2 = I_3 + I_4 + I_5$$

$$\therefore 12 = (5 + 7) = (2 + 6 + 4)$$

# Example No2:

- Find the currents flowing around the following circuit using Kirchhoff's Current Law:




- $I_T$ is the total current flowing around the circuit driven by the 12V supply voltage. At point A, $I_1$ is equal to $I_T$, thus there will be an $I_1$ *R voltage drop across resistor $R_1$

# Continued:

The circuit has 2 branches, 3 nodes (B, C and D) and 2 independent loops, thus the I*R voltage drops around the two loops will be:

$$\text{Loop ABC} \Rightarrow 12 = 4I_1 + 6I_2$$

$$\text{Loop ABD} \Rightarrow 12 = 4I_1 + 12I_3$$

Since Kirchhoff's current law states that at node B, $I_1 = I_2 + I_3$, we can therefore substitute current $I_1$ for $(I_2 + I_3)$ in both of the following loop equations and then simplify.

## Kirchhoff's Loop Equations

# Continued:

Loop (ABC)

$$12 = 4I_1 + 6I_2$$

$$12 = 4(I_2 + I_3) + 6I_2$$

$$12 = 4I_2 + 4I_3 + 6I_2$$

$$12 = 10I_2 + 4I_3$$

Loop (ABD)

$$12 = 4I_1 + 12I_3$$

$$12 = 4(I_2 + I_3) + 12I_3$$

$$12 = 4I_2 + 4I_3 + 12I_3$$

$$12 = 4I_2 + 16I_3$$

# Continued:

We now have two simultaneous equations that relate to the currents flowing around the circuit.

Eq. No 1 : $12 = 10I_2 + 4I_3$

Eq. No 2 : $12 = 4I_2 + 16I_3$

By multiplying the first equation (Loop ABC) by 4 and subtracting Loop ABD from Loop ABC, we can be reduced both equations to give us the values of $I_2$ and $I_3$

# Continued:

Eq. No 1 : $12 = 10I_2 + 4I_3 (x4) \Rightarrow 48 = 40I_2 + 16I_3$

Eq. No 2 : $12 = 4I_2 + 16I_3 (x1) \Rightarrow 12 = 4I_2 + 16I_3$

Eq. No 1 – Eq. No 2 $\Rightarrow 36 = 36I_2 + 0$

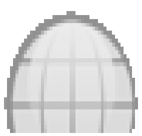
Substitution of $I_2$ in terms of $I_3$ gives us the value of $I_2$ as 1.0 Amps

Substitution of $I_3$ in terms of $I_2$ gives us the value of $I_3$ as 0.5 Amps

As Kirchhoff's junction rule states that : $I_1 = I_2 + I_3$

The supply current flowing through resistor $R_1$ is given as : $1.0 + 0.5 = 1.5$ Amps

Thus $I_1 = I_T = 1.5$ Amps, $I_2 = 1.0$ Amps and $I_3 = 0.5$ Amps.



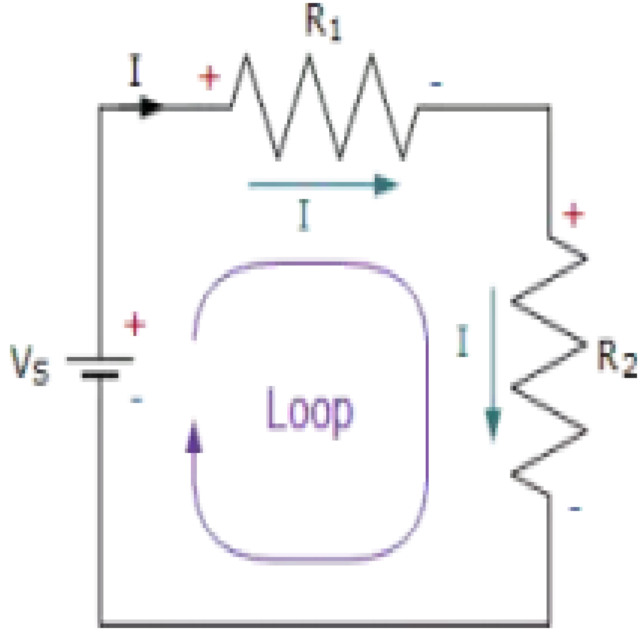
## 2- Kirchhoff's Voltage Law

His voltage law states that for a closed loop series path the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero

In other words:

$$\Sigma V = 0$$

This idea by Kirchhoff is commonly known as the Conservation of Energy, as moving around a closed loop, or circuit, you will end up back to where you started in the circuit and therefore back to the same initial potential with no loss of voltage around the loop.



# Example No1:

- Three resistor of values: 10 ohms, 20 ohms and 30 ohms, respectively are connected in series across a 12 volt battery supply. Calculate: a) the total resistance, b) the circuit current, c) the current through each resistor, d) the voltage drop across each resistor, e) verify that Kirchhoff's voltage law, KVL holds true.

# Solution:

## a) Total Resistance ($R_T$)

$$R_T = R_1 + R_2 + R_3 = 10\Omega + 20\Omega + 30\Omega = 60\Omega$$

Then the total circuit resistance $R_T$ is equal to 60Ω

## b) Circuit Current (I)

$$I = \frac{V_S}{R_T} = \frac{12}{60} = 0.2A$$

Thus the total circuit current I is equal to 0.2 amperes or 200mA

## c) Current Through Each Resistor

The resistors are wired together in series, they are all part of the same loop and therefore each experience the same amount of current. Thus:

$$I_{R1} = I_{R2} = I_{R3} = I_{SERIES} = 0.2 \text{ amperes}$$

# Continued:

**d) Voltage Drop Across Each Resistor**

$$V_{R1} = I \times R_1 = 0.2 \times 10 = 2 \text{ volts}$$

$$V_{R2} = I \times R_2 = 0.2 \times 20 = 4 \text{ volts}$$

$$V_{R3} = I \times R_3 = 0.2 \times 30 = 6 \text{ volts}$$

**e) Verify Kirchhoff's Voltage Law**

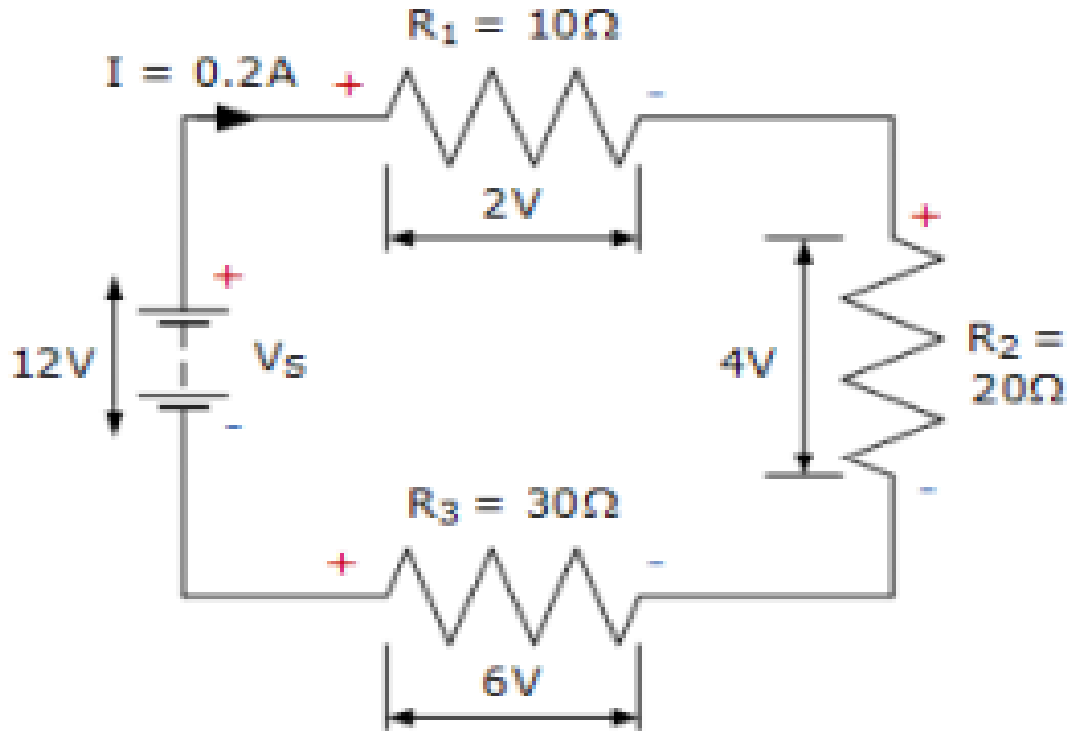
$$V_S + (-IR_1) + (-IR_2) + (-IR_3) = 0$$

$$12 + (-0.2 \times 10) + (-0.2 \times 20) + (-0.2 \times 30) = 0$$

$$12 + (-2) + (-4) + (-6) = 0$$

$$\therefore 12 - 2 - 4 - 6 = 0$$

# Continued:



# For more example:

- **https://youtu.be/Z2QDXjG2ynU**

# Assignment:

Use Kirchhoff voltage law to find (v1) and (v2):