PROBABILITY THEORY

Probability theory is the deductive part of statistics. Its purpose is to give a precise mathematical definition or structure to what has so far been an intuitive concept of randomness. Defining randomness will allow us to make exact probability statements. For example when discussing association, we could only make rough statements in terms of tendencies.

Mathematically, probability is a set function which means it is premised on sets. Therefore, we are beginning this discussion by explaining the fundamental nature of sets and the basic operations performed on sets and elements as the main ideas behind the probability function.

- **General Notions of the Probability Theory**

1- **Definition of a Set -** set A is a collection of elements. Elements are basic intuitive mathematically undefined entities. To define a set, it is necessary to be able to determine whether any element is included or not included in the set. The notion of inclusion is also an intuitive undefined concept.

2- **Definition of Elementary Events -** In probability theory, the probability assumptions are based on elements that are part of a set. The sets' elements are called elementary events. In practice, these elementary events may be measurable by units, cases, samples, points, etc.

Example:

{heads or tails} – when tossing a coin

{1,2,3,4,5,6} – when throwing a dice

The set of all results will be denoted by Ω and called ***sample space*** (of the elementary events). The elementary event $\{\omega\}$ is a subset of the Ω set which contains one element ω from Ω set, $\omega \in \Omega$.

Then the event $\boldsymbol{A}$ will be an arbitrary subset of Ω, $\boldsymbol{A} \subset \Omega$.

From statistical data we can easily establish that share of boys born in particular years with respect to all born children is moving around 51.5%. Despite the fact that in individual cases we can't predict sex of a child we can make a relatively accurate guess about how many boys there are among 10 000 children.

As the example suggests, relative frequencies of some events are stabilized with increased repetition of certain values. We shall call this phenomenon *Stability of the Relative*

Frequencies and it is is an empirical fundamental principle of the probability theory. **Relative frequency** is number n(A)/n where n is a total number of random observations and n(A) is a number of observations with an A result.

Summary:

Probability theory is a mathematical branch using axiomatic logical structure.

Mathematical statistics is a science concerned with data mining, data analysis, and formulating results

Random observation is every finite process where the result is not set by conditions under which it is run.

Sample space Ω is a set of all possible outcomes of an a random observation.

Relative frequencies of some events with increased repetition indicate some level of ***stability.***

THE TYPES OF ELEMENTARY EVENTS

If an elementary event $\omega \in \Omega$ ($\omega \in$ A) occurs then you can say that an even A has occured.

This result is denoted by $\omega \in$ A and is **favorable to the event A.**

I] **Certain event**

Certain event is the event which occurs with each random experiment. It is equivalent to the Ω set.

The certain event for example occurs when you throw a dice and you end up with one of the six numbers: 1,2,3,4,5,6.

2] Impossible event

Impossible event is the event which never occurs in an experiment. It will be denoted by **Φ**.

The impossible event for example would be throwing number 8 (with the same dice

RELATIONS BETWEEN EVENTS

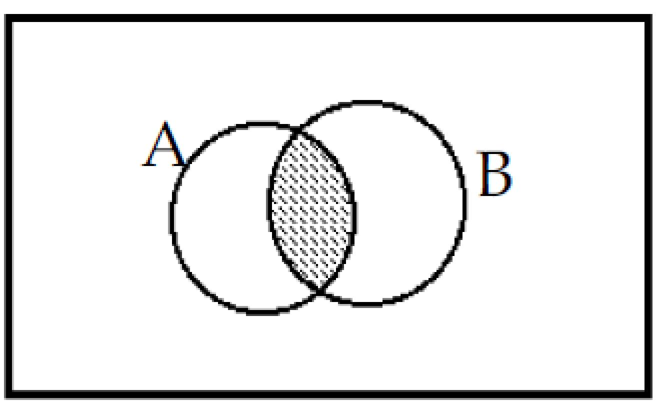
Operations on Sets - The operations of union, intersection, complementation (negation), subtraction, the concept of subset, and the null set and universal set or sample space are the algebra of sets.

1] Intersection A ∩ B

Intersection A ∩ B is the set of all elements that are both in A and in B.

Graphical example:

$$A \bigcap B = \left\{ \omega \,\middle|\, \omega \in A \wedge \omega \in \mathrm{B} \right\}$$



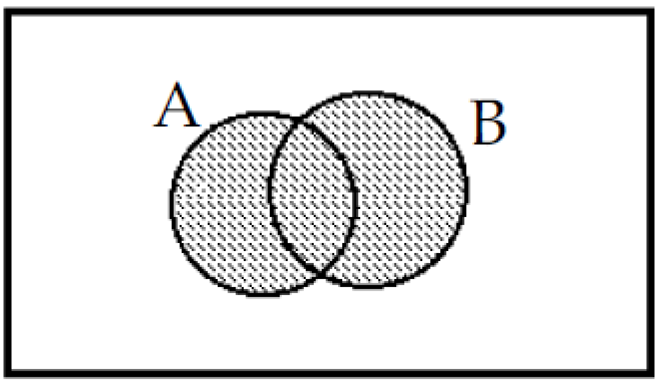
Example for throwing a dice: Numbers 2, 3 or 4 are thrown as event A and an even number is thrown as event B. It is obvious that A Ç B ={2,4}.

2] Union A U ***B***

Union A U B it is the set of all elements that are either in A or in B.

Graphical example:

$$A \bigcup B = \{\omega \mid \omega \in A \vee \omega \in B\}$$



Example – throwing a dice: Event A = {1,3,4} and event B is an even number. It's obvious that A U B ={1,2,3,4,6}.

3] Disjoint events A ∩ B = Φ

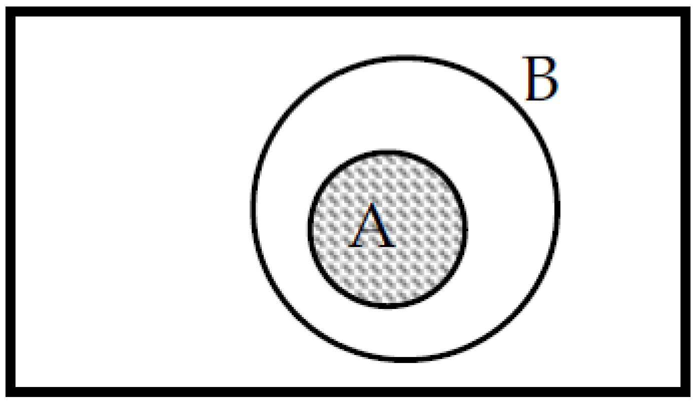
Two events A and B can't occur together. They have no common result. Example for throwing a dice: You throw an even number as event **A** and an odd number as event **B**. These events never have the same result. If event A occurs then event B can't happen.

4**]** ***Subsets (Subevent)*** **A *C* B**

A is a subset of B if each element of A is also an element of B. It means that if event A occurs than event B occurs as well.

Graphical example:

$$A \subset B \Leftrightarrow \{\omega \in \mathrm{A} \Rightarrow \omega \in B\}$$



Example for throwing a dice: You throw number 2 as event A and you throw an even number as event B. The event A is subevent of event B.5] **Events A and B are equivalent A** = **B** if **A *C* B** and at the same time **B *C* A**.

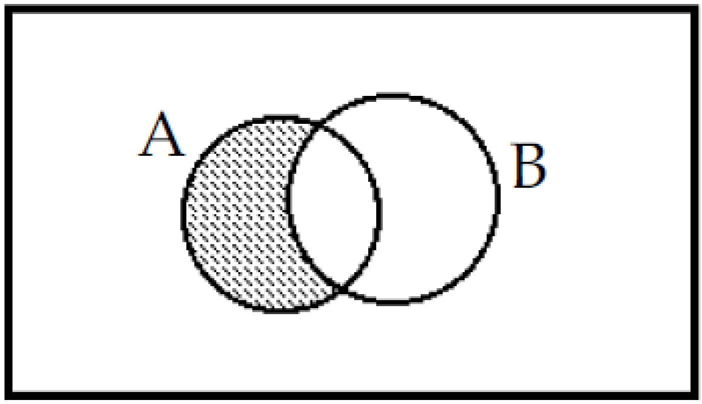
Example for throwing a dice: You throw an even number as event A and you throw a number that is dividable by number 2 as event B. These events are equivalent.

6] ***Subtraction A-B***

The set of all elements that are in A but not in B

$$A - \mathrm{B} = \mathrm{A} \cap \overline{\mathrm{B}}$$
$$A - \mathrm{B} = \{\omega \,|\, \omega \in \mathrm{A} \wedge \omega \notin \mathrm{B}\}$$

Graphical example:



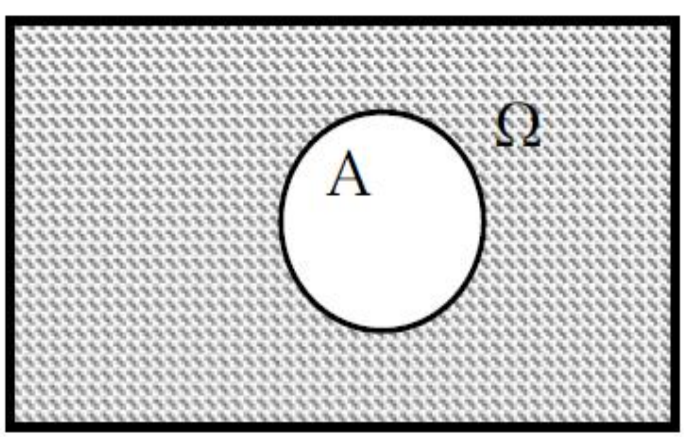
Example for throwing a dice: You throw a number greater than 2 as event A and you throw an even number as event B. Subtraction of the two events A – B = {3,5}.

7] ***Complement of event A (opposite event)***

The set of all elements that are not in A.

$$\overline{A} = \{\omega \mid \omega \notin A\}$$

Graphical example:



Example for throwing a dice: You throw an even number as event A and you throw an odd number as event $\overline{A}$

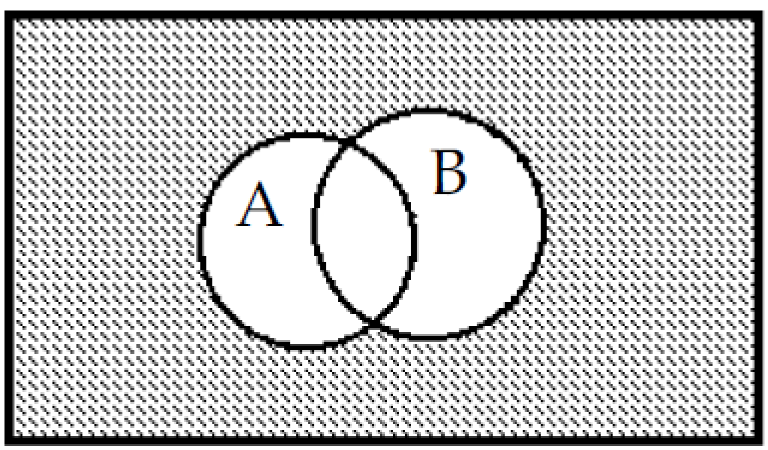
DeMorgan's Laws

- DeMorgan's Laws are logical conclusions of the fundamental concepts and basic operations of the set theory.

- ***Law no. 1***

The set of all elements that are neither in A nor in B.

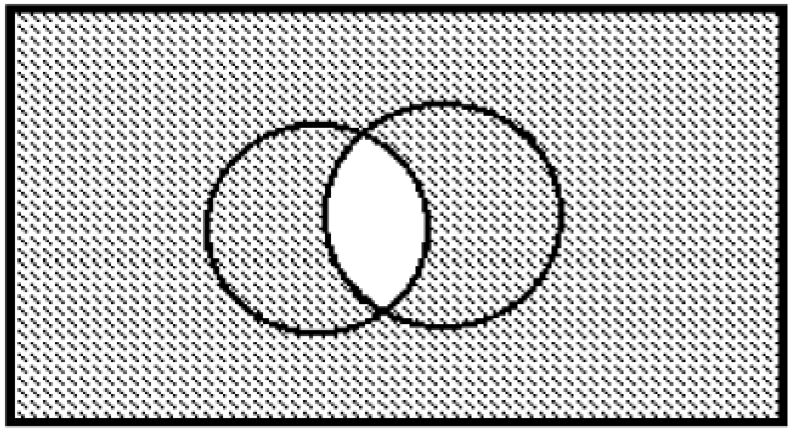
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$



- ***Law no. 2***

The set of all elements that are either not in A or not in B (that are not in the intersection of A and B).

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$



8] ***Mutually disjoint sets and partitioning the sample space***

The collection of sets $\{A_1, A_2, A_3, \ldots\}$ partition the sample space Ω:

$A_i \cap A_j = \Phi$ for

$i \neq j$

$$\Omega = \bigcup_{i=1}^{n} A_i$$