NUSU

Faculty of Administrative Sciences

Business Mathematics-2   Semester: 2

LC(9)

Integrals

Antiderivatives

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all in I

Indefinite Integrals

The collection of all antiderivatives of f is called the indefinite integral of f with respect to x, and is denoted by

$$\int f(x)\,dx$$

The symbol $\int$ is an integral sign. The function f is the integrand of the integral, and x is the variable of integration

Basic Integration Rules

The inverse nature of integration and differentiation can be verified by substituting $F'(x) = f(x)$ in the indefinite integration definition to obtain

$$\int F'(x)\,d = F(x) + c$$

Moreover, if $\int f(x)\,dx = F(x) + c$ then

$$\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x) + c$$

These two equations allow you to obtain integration formulas directly from differentiation formulas, as shown in the following summary.

Differentiation Formula	Integration Formula
$\frac{d}{dx}(c) = 0$	$\int 0\,dx = c$
$\frac{d}{dx}(kx) = k$	$\int x\,dx = kx + c$
$\frac{d}{dx}(kf(x)) = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$
$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$	$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}\big(sin(ax)\big) = acos(ax)$	$\int cos(ax)\,dx = \frac{1}{a}sin(ax) + c$
$\frac{d}{dx}\big(cos(ax)\big) = -asin(ax)$	$\int sin(ax)\,dx = -\frac{1}{a}cos(ax) + c$
$\frac{d}{dx}\big(tan(ax)\big) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}tan(ax) + c$
$\frac{d}{dx}\big(cot(ax)\big) = -acosec^2(ax)$	$\int cosec^2(ax)\,dx = -\frac{1}{a}cot(ax) + c$
$\frac{d}{dx}\big(sec(ax)\big) = asec(ax).tan(ax)$	$\int sec(ax).tan(ax)\,dx = \frac{1}{a}sec(ax) + c$
$\frac{d}{dx}\big(cosec(ax)\big) = -acosec(ax).cot(ax)$	$\int cosec(ax).cot(ax)\,dx = -\frac{1}{a}cosec(ax) + c$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$

Example 1

Find the integral

$$\int 2\,dx$$

Solution

$$\int 2\,dx = 2x + c$$

Example 2

Find the integral

$$\int 3x\,dx$$

Solution

$$\int 3x\,dx = 3\int x\,dx = 3\left(\frac{x^2}{2}\right) + c$$

Example3

Find the integral

$$\int x^2\,dx$$

Solution

$$\int x^2\,dx = \frac{1}{3}x^3 + c$$

Example 4

Find the integral

$$\int (x+3)\,dx$$

Solution

$$\int (x+3)\,dx =$$

$$\int x\,dx + \int 3\,dx = \frac{1}{2}x^2 + 3x + c$$

Example 5

Find the integral

$\int (x+1)^2\,dx$

Solution

$$\int (x+1)^2\,dx =$$

$$\int (x^2 + 2x + 1)\,dx = \int x^2\,dx + \int 2x\,dx + \int 1\,dx$$
$$= \frac{1}{3}x^3 + \frac{2}{2}x^2 + x + c = \frac{1}{3}x^3 + x^2 + x + c$$

Example 6

Find the integral

$\int sin4x\, dx$

Solution

$\int sin4x\, dx = -\frac{1}{4}cos4x + c$

Example 7

Find the integral

$\int \sec^2 x\, dx$

Solution

$$\int \sec^2 x\, dx = tanx + c$$

Example 8

Find the integral

$\int cosec2x.\, cot2x\, dx$

Solution

$$\int cosec2x.\, cot2x\, dx = cosec2x + c$$

Example 9

Find the integral

$\int (cos2x + \sec^2 3x)\, dx$

Solution

$$\int (cos2x + \sec^2 3x)\, dx = \int cos2x\, dx + \int \sec^2 3x\, dx$$
$$= \frac{1}{2} sin2x + \frac{1}{3} tan3x + c$$

Example 10

Find the integral

$\int e^{3x}\, dx$

Solution

$$\int e^{3x}\, dx = \frac{1}{3} e^{3x} + c$$

Example 11

Find the integral

$\int (x + 3 + e^x)\, dx$

Solution

$$\int (x + 2 - e^x)\, dx = \int x\, dx + \int 3\, dx - \int e^x\, dx$$
$$= \frac{1}{2} x^2 + 3x - e^x + c$$

Tut (9)

Find the integral of the following functions

1. $\int (x^3 + 5x - 3)\, dx$
2. $\int (x - 2)^2\, dx$
3. $\int 9\, dx$
4. $\int sin2x\, dx$
5. $\int (cosx - 3sinx)\, dx$
6. $\int \sec x\, (\sec x + tanx)\, dx$
7. $\int e^{5x}\, dx$
8. $\int (x^2 + cosec^2 x)\, dx$
9. $\int (x + e^x + sinx)\, dx$