NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

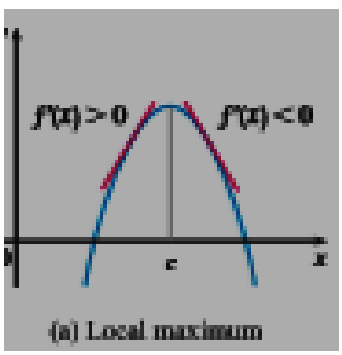
LC(8)

Limits Function of Maxima and Minima

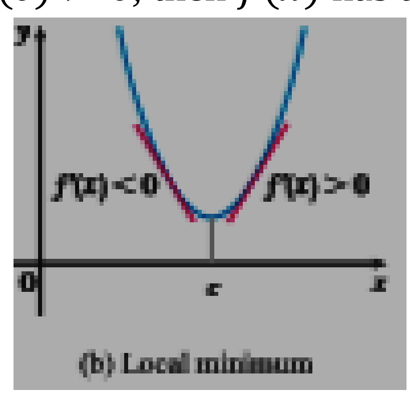
The Second Derivative Test

Suppose that c is a critical number of a continuous function (x) .

(a) If $f''(c) <$, then $f(x)$ has a local maximum at c .



(b) If $f''(c) > 0$, then $f(x)$ has a local minimum at c.



- **Critical number c** of a functions $f(x)$ is value of x such satisfy the condition $f'(x) = 0$

For example to find the critical number of $f(x) = x^2 - 4x + 3$

we found the derivative of $f(x)$ is

$$f'(x) = 2x - 4$$

And found the value of x such satisfy the condition

$$f'(x) = 2x - 4 = 0$$

Then the value of x is $x = 2$

EXAMPLE 1

Find the local maximum and minimum values of the function

$$f(x) = 2x^2 + 4x - 1$$

SOLUTION

To find the critical numbers of (x) , we differentiate:

$$f'(x) = 4x + 4$$

So $f'(x) = 4x + 4 = 0$ The critical numbers of this equation is $x = -1$

To find the local maximum and minimum values, we find the second derivative $f''(x) = 4$

Such that

$f''(-1) = 4 > 0$ then the function $f(x)$ is the local minimum values in $x = 4$

EXAMPLE 2

Find the local maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 - 1$$

SOLUTION

To find the critical numbers of (x) , we differentiate:

$$f'(x) = 3x^2 - 6x$$

So $f'(x) = 3x^2 - 6x = 0$ The critical numbers of this equation is $x_1 = 0$ and $x_2 = 2$

To find the local maximum and minimum values, we find the second derivative $f''(x) = 6x - 6$

Such that

$f''(0) = -6 < 0$ then the function $f(x)$ is the local maximum values in $x_1 = 0$

$f''(2) = 6 > 0$ then the function $f(x)$ is the local minimum values in $x_2 = 2$

EXAMPLE 3

Find the local maximum and minimum values of the function

$$f(x) = x^3 - 6x^2 - 15x - 5$$

SOLUTION

To find the critical numbers of (x) , we differentiate:

$$f'(x) = 3x^2 - 12x - 15$$

So $f'(x) = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x + 1)(x - 5) = 0$
The critical numbers of this equation is $x_1 = -1$ and $x_2 = 5$

To find the local maximum and minimum values, we find the second derivative $f''(x) = 6x - 12$

Such that

$f''(-1) = -18 < 0$ then the function $f(x)$ is the local maximum values in $x_1 = -1$

$f''(5) = 18 > 0$ then the function $f(x)$ is the local minimum values in $x_2 = 5$

Tut(8)

Find the local maximum and minimum values of this function

A) $f(x) = x^2 + 10$

B) $f(x) = 2x^2 - 4x$

C) $f(x) = 3x^2 + 5x$

D) $f(x) = x^2$

E) $f(x) = x^3 - 5$

F) $f(x) = x^3 - 4x$

G) $f(x) = x^3 + 2x^2$

H) $f(x) = x^3 - 9x^2 - 8$

I) $f(x) = 2x^3 + 3x^2 + 12x$