NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(7)

Higher-Order Derivatives

Just as you can obtain a velocity function by differentiating a position function, you can obtain an acceleration function by differentiating a velocity function. Another way of looking at this is that you can obtain an acceleration function by differentiating a position function twice

$s(t)$ Position function

$v(t) = s'(t)$ Velocity function

$a(t) = v'(t) = s''(t)$ Acceleration function

The function given by $a(t)$ is the second derivative of $s(t)$ and is denoted by $s''(t)$

The second derivative is an example of a higher-order derivative. You can define derivatives of any positive integer order. For instance, the third derivative is the derivative of the second derivative.

NOTE

The second derivative of f is the derivative of the first derivative of f.

Higher-order derivatives are denoted as follows.

First derivative: y' , $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$

Second derivative: y' , $f''(x)$, $\frac{d^2y}{dx^2}$, $\frac{d^2}{dx^2}[f(x)]$

Third derivative: y' , $f'''(x)$, $\frac{d^2y}{dx^2}$, $\frac{d^2}{dx^2}[f(x)]$

Fourth derivative: $y^{(4)}$, $f^{(4)}(x)$, $\frac{d^{(4)}y}{dx^{(4)}}$, $\frac{d^{(4)}}{dx^{(4)}}[f(x)]$

⋮

nth derivative: $y^{(n)}$, $f^{(n)}(x)$, $\frac{d^{(n)}y}{dx^{(n)}}$, $\frac{d^{(n)}}{dx^{(n)}}[f(x)]$

EXAMPLE 1

Find the acceleration of the position function given by

$$s(t) = -0.7t^2 - 1$$

Solution

To find the acceleration, differentiate the position function twice.

$s(t) = -0.7t^2 - 1$ Position function

$s'(t) = -1.4t$ Velocity function

$s''(t) = -1.4$ Acceleration function

EXAMPLE 2

Find the second derivative of the given function.

$\boldsymbol{a})\ f(x) = x^3 - 3x^2 - 2x + 5$

$\boldsymbol{b})\ f(x) = x^2 cosx$

Solution

$\boldsymbol{a})\ f(x) = x^3 - 3x^2 - 2x + 5$

$f'(x) = 3x^2 - 6x - 2$

$f''(x) = 6x - 6$

$\boldsymbol{b})\ f(x) = x^2 cosx$

$f'(x) = -x^2 sinx + 2xcosx$

$f''(x) = -x^2 cosx - 2xsinx - 2xsinx + 2cosx$

$= -x^2 cosx - 4xsinx + 2cosx$

EXAMPLE 3

Find the given higher-order derivative.

1) $f(x) = x^5$ $f^{(4)}(x)$?

2) $f(x) = sin3x$ $f'''(x)$?

3) $f(x) = \dfrac{x}{cosx}$ $f''(x)$?

Solution

1) $f(x) = x^5$

$f'(x) = 5x^4$

$f''(x) = 20x^3$

$f'''(x) = 60x^2$

$f^{(4)}(x) = 120x$

2) $f(x) = sin3x$

$f'(x) = 3cos3x$

$f''(x) = -9sin3x$

$f'''(x) = -27cos3x$

3) $f(x) = \frac{x}{cosx}$

$f'(x) = \frac{cosx + xsinx}{(cosx)^2}$

$f''(x)$

$= \frac{(cosx)^2(-sinx + xcosx + sinx) - (cosx + xsinx)(-2cosxsinx)}{(cosx)^4}$

Tut(7)

1- Find the acceleration of the position function given by

$$s(t) = -0.3t^2 + 2$$

2- Find the second derivative of the given function.

$\boldsymbol{a}$) $f(x) = 2x^5 + x^2 - 8x - 1$

$\boldsymbol{b}$) $f(x) = xsinx$

$\boldsymbol{c}$) $f(x) = \dfrac{x^2 + 5x}{6x}$

3- Find the given higher-order derivative.

1) $f(x) = x^5$ $\quad f^{(5)}(x)$?

2) $f(x) = e^{2x}$ $\quad f'''(x)$?

3) $f(x) = \dfrac{x + 1}{sinx}$ $\quad f''(x)$?