NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(4)

The Derivative

Definition

The function f' defined by the formula

$$f'(x) = \lim_{\nabla x \to 0} \frac{f(x+\nabla x) - f(x)}{\nabla x}$$

Is called the **derivative of f with respect to x**. The domain of f' consists of all x in the domain of f for which the limit exists

Example 1(By using definition)

Find the derivative with respect to x of $f(x) = x^2$, at $x = 2$

Solution.

It follows that

$$f'(x) = \lim_{\nabla x \to 0} \frac{f(x+\nabla x) - f(x)}{\nabla x} = \lim_{\nabla x \to 0} \frac{(x+\nabla x)^2 - (x)^2}{\nabla x}$$

$$= \lim_{\nabla x \to 0} \frac{x^2 + 2x\nabla x + (\nabla x)^2 - x^2}{\nabla x} = \lim_{\nabla x \to 0} \frac{2x\nabla x + (\nabla x)^2}{\nabla x}$$

$$= \lim_{\nabla x \to 0} \frac{\nabla x(2x + \nabla x)}{\nabla x} = \lim_{\nabla x \to 0} 2x + \nabla x = 2x$$

$$\text{at } x = 2 \text{ then } f'(x)|_{x=2} = 2(2) = 4$$

Example 2 (By using definition)

Find the derivative with respect to x of $f(x) = x^3 - x$.

Solution

$$f'(x) = \lim_{\nabla x \to 0} \frac{f(x+\nabla x) - f(x)}{\nabla x} = \lim_{\nabla x \to 0} \frac{(x+\nabla x)^3 - (x+\nabla x) - (x^3 - x)}{\nabla x}$$

$$= \lim_{\nabla x \to 0} \frac{x^3 + 3x^2\nabla x + 3x(\nabla x)^2 + (\nabla x)^3 - x - \nabla x - x^3 + x}{\nabla x}$$

$$= \lim_{\nabla x \to 0} \frac{3x^2\nabla x + 3x(\nabla x)^2 + (\nabla x)^3 - \nabla x}{\nabla x}$$

$$= \lim_{\nabla x \to 0} \frac{\nabla x(3x^2 + 3x\nabla x + (\nabla x)^2 - 1)}{\nabla x}$$

$$= \lim_{\nabla x \to 0} 3x^2 + 3x\nabla x + (\nabla x)^2 - 1 = 3x^2 - 1$$

OTHER DERIVATIVE NOTATIONS

The process of finding a derivative is called differentiation. You can think of differentiation as an operation on functions that associates a function f' with a function f. When the independent variable is x, the differentiation operation is also commonly denoted by

$$f'(x) = \frac{d}{dx}[f(x)]$$

In the case where there is a dependent variable $y = f(x)$, the derivative is also commonly denoted by

$$f'(x) = y'(x) \qquad \text{or} \qquad f'(x) = \frac{dy}{dx}$$

DERIVATIVE BY USING ROLES

In this case we will develop some important theorems that will enable us to calculate derivatives more efficiently

DERIVATIVE OF A CONSTAN

Theorem

The derivative of a constant function is 0; that is, if c is any real number, then

$$\frac{d}{dx}[c] = 0$$

Example

solve

$$\frac{d}{dx}[1] = 0 \quad , \quad \frac{d}{dx}[-3] = 0 \quad , \quad \frac{d}{dx}[\pi] = \ 0 \, , \quad \frac{d}{dx}\left[-\sqrt{2}\right] = 0$$

DERIVATIVES OF POWER FUNCTIONS

Theorem (The Power Rule)

If n is a positive integer, then

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Example

solve

$$\frac{d}{dx}[x] = 1 \quad , \quad \frac{d}{dx}[x^4] = 4x^3 \quad , \quad \frac{d}{dx}[x^5] = 5x^4 \quad , \qquad \frac{d}{dt}[t^{12}] = 12t^{11}$$

Theorem (Constant Multiple Rule)

If f is differentiable at x and c is any real number, then cf is also differentiable at x and

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Example

solve

1) $\frac{d}{dx}[4x^8] = 4\frac{d}{dx}[x^8] = 4[8x^7] = 32x^7$

2) $\frac{d}{dx}[-x^{12}] = -1\frac{d}{dx}[x^{12}] = -12x^{11}$

3) $\frac{d}{dx}\left[\frac{\pi}{x}\right] = \pi\frac{d}{dx}[x^{-1}] = \pi(-x^{-2}) = -\frac{\pi}{x^2}$

DERIVATIVES OF SUMS AND DIFFERENCES

Theorem (Sum and Difference Rules)

If f and g are differentiable at x, then so are $f + g$ and $f - g$ an

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$$

Example

solve

1) $\frac{d}{dx}[2x^6 + x^{-9}] = \frac{d}{dx}[2x^6] + \frac{d}{dx}[x^{-9}] = 12x^5 + (-9)x^{-10}$
$= 12x^5 - 9x^{-10}$

2) $\frac{d}{dx}\left[\frac{\sqrt{x} - 2x}{\sqrt{x}}\right] = \frac{d}{dx}\left[1 - 2\sqrt{x}\right] = \frac{d}{dx}[1] - \frac{d}{dx}[2\sqrt{x}] = 0 - 2(\frac{1}{2\sqrt{x}})$
$= -\frac{1}{\sqrt{x}}$

Tut(5)

A) Find $f'(x)$ by using the definition

1) $f(x) = 5$
2) $f(x) = x - 2$
3) $f(x) = x^3$
4) $f(x) = x^2 - 2x + 3$

B) Find $\frac{dy}{dx}$ by

a) $y = 4$
b) $y = 3x$
c) $y = x^7$
d) $y = 2x^4 - 4x^2$
e) $y = 3x^3 + 2x^2 - 6x + 4$
f) $y = 2x^{-3} + \frac{7}{x^2} + 2$