NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(3)

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

IRRATIONAL EXPONENTS

Recall from algebra that if b is a nonzero real number, then nonzero integer powers of b are defined by

$$\mathrm{b}^{\mathrm{n}} = \underbrace{\mathrm{b} \times \mathrm{b} \times \cdots \times \mathrm{b}}_{\text{n factors}} \text{ and } \mathrm{b}^{-\mathrm{n}} = \frac{1}{\mathrm{b}^{\mathrm{n}}}$$

and if $n = 0$, then $b^0 = 1$. Also, if p/q is a positive rational number expressed in lowest terms, then

$$b^{\frac{p}{q}} = \sqrt[q]{b^p} = \left(\sqrt[q]{b}\right)^p \text{ and } b^{-\frac{p}{q}} = \frac{1}{b^{\frac{p}{q}}}$$

With this notion for irrational powers , were mark without proof that the following familiar laws of exponents hold for all real values of p and q:

$$b^p b^q = b^{p+q} \quad , \quad \frac{b^p}{b^q} = b^{p-q} \quad , \quad (b^p)^q = b^{pq}$$

THE FAMILY OF EXPONENTIAL FUNCTIONS

A function of the form $f(x) = b^x$, where $b > 0$, is called an exponential function with base b. Some examples are

$$f(x) = 2^x \,, \quad f(x) = \left(\frac{1}{2}\right)^x \quad , \quad f(x) = \pi^x$$

Notet hat anexponential function has aconstant base and variable exponent. Thus,functions such as $f(x) = x^2$ and $f(x) = x^\pi$ would not be classified as exponential functions, since they have a variable base and a constant exponent

The domain and range of the exponential function $f(x) = b^x$ can also be found by

• If $b > 0$, then $f(x) = b^x$ is defined and has a real value for every real value of x, so the **natural domain of every exponential function is** $(-\infty, +\infty)$.

• If $b > 0$ and $b \neq 1$, then as noted earlier the graph of $y\ = b^x$ increases indefinitely as it is traversed in one direction and decreases toward zero but never reaches zero as it is traversed in the other direction. This implies that the range of $f(x) = b^x$ is $(0, +\infty)$

LOGARITHMIC FUNCTIONS

Recall from algebra that a logarithm is an exponent. More precisely, if $b > 0$ and $b \neq 1$, then for a positive value of x the expression

$$\log_b x$$

(read "the logarithm to the base b of x") denotes that exponent to which b must be raised to produce x. Thus, for example,

$$\log_{10} 100 = 2\ ,\quad \log_{10} \frac{1}{1000} = -3\ ,\ \log_2 16 = 4\ ,log2\ 16 = 4\ ,$$
$$\log_b 1 = 0 \quad ,\log_b b = 1$$

We call the function $f(x) = \log_b x$ **the logarithmic function with base b**.

Logarithms with base 10 are called **common logarithms** and are often written without explicit reference to the base. Thus, the symbol $\log x$ generally denotes $\log_{10} x$.

Logarithmic functions can also be viewed as inverses of exponential functions. that if $b > 0$ and $b \neq 1$, then the graph of $f(x) = b^x$ passes the horizontal line test, so b^x has an inverse. We can find a formula for this inverse with x as the independent variable by solving the equation

$$x = b^y$$

for y as a function of x. But this equation states that y is the logarithm to the base b of x, so it can be rewritten as $y = \log_b x$

The most important logarithms in applications are those with base e. These are called **natural logarithms** because the function $\log_e x$ is the inverse of the **natural exponential function** e^x. It is standard to denote the natural logarithm of x by $\ln x$ (read "ell en of x"), rather than $\log_e x$. For example

$\ln 1 = 0$	$, \ln e = 1$	$, \ln(1/e) = -1,$	$\ln(e^2) = 2$
Since $e^0 = 1$	Since $e^1 = e$	Since $e^{-1} = 1/e$	Since $e^2 = e^2$

In general,

$$y = \ln x \quad \text{if and only if} \quad x = e^y$$

Trigonometric functions

The function of the form $y = sinx$ and $y = cosx$ is said to be trigonometric functions

$tanx$	$\frac{sinx}{cosx}$
$cotx$	$\frac{cosx}{sinx}$
$secx$	$\frac{1}{cosx}$
$cosecx = cscx$	$\frac{1}{sinx}$