NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(2)

The domain and the range of functions

The set A is called the domain of the function f, and the set of all the possible values of $f(x)$ is called the range of the function f.

The variable x is called the independent variable and the variable $y = f(x)$ is called the dependent variable.

Example 1

Determine the domain and the range for each of the following functions:

1) $f(x) = x^2 + 2x + 3$
2) $f(x) = \frac{1}{x-1}$
3) $f(x) = \sqrt{x + 1}$

Solution

1) **This function is a polynomial function, and the domain of every polynomial function is** $\mathbb{R}$ (the set of all the real numbers $(-\infty, \infty)$). The range of every polynomial is also the set $\mathbb{R}$, because if we substitute the set of all the real number we will get the set of all the real number.
2) **This is a rational function, and division by 0 is not allowed**, so this function is not defined when $x = 1$, so the domain of the function f is the set of all real number except $x = 1$, which could also be written in interval notation as $\mathbb{R} - \{1\}$. The range of this

function is the set of all real numbers except 0, as this function can never equal to 0 for any value of x, which could also be written in interval notation as $\mathbb{R} - \{0\}$.

3) **This is the root function and we know the root is only defined for the non negative real numbers**, so the domain is defined by the interval $[-1, \infty)$. The range of this function is also the set of the non negative real number, also it can be written in the interval notation $[0, \infty)$

Even and Odd functions

Definition

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an **even function**.

For instance, the function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

If a function f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**.

For instance, the function $f(x) = x^3$ is even because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

Example 3

Determine whether each of the following functions is even, odd, or neither even nor odd.

a) $f(x) = x^5 + x$
b) $g(x) = 1 - x^4$
c) $h(x) = 2x - x^2$

Solution

a)

$$f(-x) = (-x)^5 + (-x) = (-1)^5 x^5 - x$$

$$= -x^5 - x = -(x^5 + x)$$

$$= -f(x).$$

Therefore, f is an odd function

b)

$g(-x) = 1 - (-x)^4 = 1 - x^4 = g(x).$

So t is even

c)

$$h(-x) = 2(-x) - (-x)^2 = -2x - x^2$$

$Since\ h(-x) \neq h(x)\ and\ h(-x) \neq -h(x),$

We conclude that h is neither even nor odd

Tut(2)

a) Determine the domain and the range for each of the following functions:

1) $f(x) = x^3 - x + 1$
2) $f(x) = x - 3$
3) $f(x) = \frac{1}{x}$
4) $f(x) = \frac{2}{x+2}$
5) $f(x) = \sqrt{x - 5}$
6) $f(x) = \sqrt{x + 1}$

b) Determine whether each of the following functions is even, odd, or neither even nor odd.

1) $f(x) = x^3 - x$
2) $f(x) = -x$
3) $f(x) = x^2 + 3$
4) $f(x) = x^4 - x^2$
5) $f(x) = 2x - x^2$
6) $f(x) = x^3 + 7$