NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(13)

Mathematical applications in business and economics

Total Revenue

The total revenue is

$$R(x) = (\text{number of units sold})(\text{price per unit})$$

Profit

The profit is $P(x) = \text{total revenue} - \text{total cost}$

$$= R(q) - C(q)$$

Average cost

The average cost is

$$A(x) = \frac{C(x)}{x}$$

Example 1

A manufacturer estimates that when x thousand units of a particular commodity are produced each month, the total cost will be $C(x) = 2x^2 - 5x + 3$ hindered dollars, and all x units can be sold at a price of $P(x) = 3x^2 - 50$ dollars per unit find them.

a. Total Revenue
b. profit
c. Average cost

Solution

a. The total revenue is

$$R(x) = (\text{number of units sold})(\text{price per unit})$$
$$= x.P(x) = x(3x^2 - 50) = 3x^3 - 50x$$

b. The profit is $P(x) = \text{total revenue} - \text{total cost}$

$$= R(q) - C(q) = (3x^3 - 50x) - (2x^2 - 5x + 3)$$
$$= 3x^3 - 2x^2 - 45x - 3$$

c. The average cost is

$$A(x) = \frac{C(x)}{x} = \frac{2x^2 - 5x + 3}{x} = 2x - 5 + \frac{3}{x}$$

Example2

Find the critical points of $f(x) = 2x^3 + 3x^2 - 12x - 7$ and use the second derivative test to classify each critical point as a relative maximum or minimum.

Solution

Since the first derivative

$f'(x) = 6x^2 + 6x - 12 = 6(x+2)(x-1)$

is zero when $x = -2$ and $x = 1$, the corresponding points $(-2,13)$ and $(1,-14)$ are the critical points of f. To test these points, compute the second derivative

$f''(x) = 12x + 6$

and evaluate it at $x = -2$ and $x = 1$. Since

$$f(-2) = -18 < 0$$

it follows that the critical point (-2, 13) is a relative maximum, and since

$$f(1) = 18 > 0$$

it follows that the critical point $(1,-14)$ is a relative minimum

Example3

A manufacturer estimates that when x thousand units of a particular commodity are produced each month, the total cost will be $C(x) = 0.4x^2 + 3x + 40$ thousand dollars, and all x units can be sold at a price of $P(x) = 22.2 - 1.2x$ dollars per unit.

a. Determine the level of production that results in maximum profit. What is the maximum profit?

b. At what level of production is the average cost per unit $A(x) = \frac{C(x)}{x}$ minimized?

c. At what level of production is the average cost equal to the marginal cost $C'(x)$?

Solution

a. The revenue is

$$R(x) = xP(x) = x(22.2 - 1.2x)$$
$$= -1.2x^2 + 22.2x$$

thousand dollars, so the profit is

$$P(x) = R(x) - C(x)$$
$$= -1.2x^2 + 22.2x - (0.4x^2 + 3x + 40)$$
$$= -1.6x^2 + 19.2x - 40$$

Thousand dollars. We have

$$P'(x) = -1.6(2x) + 19.2 = -3.2x + 19.2 = 0$$

When

$$-3.2x + 19.2 = 0$$

$$x = \frac{19.2}{3.2} = 6$$

Since x

$P''(x) = -3.2$, it follows that $P''(6) = -3.2$, and the second derivative test tells us that maximum profit occurs when $x = 6$ (thousand) units are produced. The maximum profit is $P(6) = -1.6(6)^2 + 19.2(6) - 40 = 17.6$ thousand dollars

b. The average cost is

$A(x) = \frac{C(x)}{x} = \frac{-0.4x^2+3x+40}{x}$ (Thousand dollars)/(thousand units)

$= 0.4x + 3 + \frac{40}{x}$ dollars /units

for $x > 0$ (the level of production cannot be negative or zero). We find

$$A'(x) = 0.4 - \frac{40}{x^2} = \frac{0.4x^2 - 40}{x^2} = 0$$

which is 0 for $x > 0$ only when $x = 10$. Since

$$A''(x) = \frac{80}{x^3} > 0 \quad \text{when } x > 0$$

it follows from the second derivative test for absolute extrema that average cost $A(x)$ is minimized when $x = 10$ (thousand) units. The minimal average cost is

$A(10) = 0.4(10) + 3 + \frac{40}{10} = 11$ dollars /units

c. The marginal cost is $C'(x) = 0.8x + 3$, and it equals average cost when

$$0.8x + 3 = 0.4x + 3 + \frac{40}{x}$$
$$0.4x = \frac{40}{x}$$
$$0.4x^2 = 40$$
$$x = 10$$

which equals the optimal level of production in part (b).

Tut(13)

1) A manufacturer estimates that when x thousand units of a particular commodity are produced each month, the total cost will be $C(x) = 2x^3 + 3x - 9$ hindered dollars, and all x units can be sold at a price of $P(x) = 2x^4 + 10$ dollars per unit find them.
   - **d.** Total Revenue
   - **e.** profit
   - **f.** Average cost
2) Find the critical points of $f(x) = x^3 + 3x^2 - 9x - 7$ and use the second derivative test to classify each critical point as a relative maximum or minimum.
3) A manufacturer estimates that when x thousand units of a particular commodity are produced each month, the total cost will be $C(x) = 0.2x^2 - 2x + 20$ thousand dollars, and all x units can be sold at a price of $P(x) = 4 - 0.8x$ dollars per unit.

a. Determine the level of production that results in maximum profit. What is the maximum profit?

b. At what level of production is the average cost per unit $A(x) = \frac{C(x)}{x}$ minimized?

c. At what level of production is the average cost equal to the marginal cost $C'(x)$?