NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(11)

Increasing Functions and Decreasing Functions

All another corollary to the Mean Value Theorem, we show that functions with positive derivatives are increasing functions and functions with negative derivatives are decreasing functions. A function that is increasing or decreasing on an interval is said to be monotonic on the interval

COROLLARY 1

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).

If $f'(x) > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.

If $f'(x) < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

EXAMPLE 1

Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and on which f is decreasing.

Solution

The function f is everywhere continuous and differentiable. The first derivative

$$f'(x) = 3x^2 - 12 = 3(x^2 - 4)$$

$$= 3(x+2)(x-2)$$

is zero at $x = -2$ and $x = 2$n. These critical points subdivide the domain of f to create nonoverlapping open intervals $(-\infty, -2), (-2,2)$, and $(2, \infty)$ on which f' is either positive or negative. We determine the sign of f' by evaluating f' at a convenient point in each subinterval. The behavior of f is determined by then applying Corollary 1 to each subinterval. The results are summarized in the following table

Intervnl	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
f' evnluated	$f'(-3) = 15$	$f'(0) = -12$	$f'(3) = 15$
Sign of f'	$+$	$-$	$+$
Behavior of f	increasing	decreasing	increasing

We used "strict" less-than inequalities to specify the intervals in the summary table for Example I. Corollary 1 says that we could use $\leq$ inequalities as well. That is, the function I in the example is increasing on $-\infty \leq x \leq -2$, decreasing on $-2 \leq x \leq 2$, and increasing on $2 \leq x \leq \infty$. We do not talk about whether a function is increasing or decreasing at a single point

First Derivative Test for Local Extrema

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right,

1. if f' changes from negative to positive at c, then f has a local minimum at c;

2. if f' changes from positive to negative at c, then f has a local maximum at c;

3. if f' does not change sign at c (that is, f' is positive on both sides of cor negative on both sides), then f has no local extremum at c.

EXAMPLE 2

Find the critical points of

$$f(x) = x^{\frac{1}{3}}(x-4) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}.$$

Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

Solution

The function f is continuous at all x since it is the product of two continuous functions, $x^{\frac{1}{3}}$and $(x-4)$. The first derivative

$$f'(x) = \frac{d}{dx}\left(x^{\frac{4}{3}} - 4x^{\frac{1}{3}}\right) = \frac{4}{3}x^{\frac{1}{3}} - \frac{4}{3}x^{\frac{-2}{3}}$$

is zero at $x = 1$ and undefined at $x = 0$. There are no endpoints in the domain, so the critical points $x = 0$ and $x = 1$ are the only places where f might have an extreme value. The critical points partition the x-axis into intervals on which f' is either positive or negative. The sign pattern of f' reveals the behavior of f between and at the critical points, as summarized in the following table.

Intervnl	$x < 0$	$0 < x < 1$	$x > 1$
Sign of f'	$-$	$-$	$+$
Behavior of f	decreasing	decreasing	increasing

Corollary 1 to the Mean Value Theorem tells us that f decreases on $(-\infty, 0]$, decreases on [0,1], and increases on $[1, \infty)$. The First Derivative Test for Local Extrema tells us that f does not have an extreme value at $x = 0$ (f' does not change sign) and that f has a local minimum at $x = 1$ (f' changes from negative to positive). The value of the local minimum is $f(1) = 1^{\frac{1}{3}}(1-4) = -3$. This is also an absolute minimum since f is decreasing on $(-\infty, 1]$ and increasing on $[1, \infty)$

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Answer the following questions about the functions

a- What are the critical points of (x) ?
b- On what intervals is $f(x)$ increasing or decreasing ?

1. $f(x) = x(x-1)$
2. $f(x) = (x-1)(x+2)$
3. $f(x) = (x-1)^2(x+2)$
4. $f(x) = (x-1)^2(x+2)^2$
5. $f'(x) = 1 - \frac{4}{x^2} \qquad x \neq 0$