NUSU

Faculty of Administrative Sciences

Business Mathematics-2 Semester: 2

LC(10)

THE DEFINITE INTEGRAL

Definition

A function f is said to be integrable on a finite closed interval $[a, b]$if the limit

$$\lim_{max\ \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$

exists and does not depend on the choice of partitions or on the choice of the points x_k^* in the subintervals. When this is the case we denote the limit by the symbol

$$\int_a^b f(x)\,dx = \lim_{max\Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$

which is called the **definite integral** of f from a to b. The numbers a and b are called the lower limit of integration and the upper limit of integration, respectively, and $f(x)$ is called the integrand.

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a)$$

The notation used for the definite integral deserves some comment. Historically, the expression “$f(x)dx$” was interpreted to be the “infinitesimal area” of a rectangle with height $f(x)$ and “infinitesimal” width dx. By “summing” these infinitesimal areas, the entire area under the curve was obtained. The integral symbol “ $\int$ ” is an “elongated s” that was used to indicate this summation. For us, the integral symbol “” and the symbol “dx” can serve as reminders that the definite integral is actually a limit of a summation as $\Delta x_k \to 0$. The sum that appearsin Definition. Is called a Riemann sum, and the definite integral is sometimes called the Riemann integral in honor of the German mathematician Bernhard Riemann who formulated many of the basic concepts of integral calculus. (The reason for the similarity in notation between the definite integral and the indefinite integral will become clear in the next section, where we will establish a link between the two types of “integration.”)

Theorem

If a function f is continuous on an interval $[a, b]$, then f is integrable on $[a, b]$, and the net signed area A between the graph of f and the interval $[a, b]$is

$$A = \int_a^b f(x)\, dx$$

Example 1

Sketch the region whose area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry

$$\int_1^4 2\, dx$$

Solution

The graph of the integrand is the horizontal line $y = 2$, so the region is a rectangle of height 2 extending over the interval from 1 to 4. Thus

$$\int_1^4 2\,dx = (area\ of\ rectangle) = 2(3) = 6$$

PROPERTIES OF THE DEFINITE INTEGRAL

1. If a is in the domain of f, we define
$$\int_a^a f(x)\,dx = 0$$
2. If f is integrable on $[a, b]$, then we define
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$
3. If f and g are integrable on $[a, b]$ and if c is a constant, then $cf, f + g$, and $f - g$ are integrable on $[a, b]$ and

   a- $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

   b- $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

   c- $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

Example 1

$$\int_1^2 2\,dx$$

Solution

$$\int_1^2 2\,dx = 2x|_1^2 = 2(2) - 2(1) = 4 - 2 = 2$$

Example 2

Find the integral

$$\int_{1}^{3} 3x\,dx$$

Solution

$$\int_{1}^{3} 3x\,dx = 3\int_{1}^{3} x\,dx = 3x^2|_1^2 = 3(9) - 3(1) = 27 - 3 = 24$$

Example3

Find the integral

$$\int_{-1}^{1} x^2\,dx$$

Solution

$$\int_{1}^{3} x^2\,dx = \frac{1}{3}x^3|_{-1}^{1} = \frac{1}{3}\big(1 - (-1)\big) = \frac{2}{3}$$

Example 4

Find the integral

$$\int_{0}^{1} (x + 3)\,dx$$

Solution

$$\int_{0}^{1} (x + 3)\,dx = \frac{1}{2}x^2 + 3x\Big|_{0}^{1} = \left(\frac{1}{2} + 3\right) - (0 + 0) = \frac{7}{2}$$

Example 5

Find the integral

$$\int_{0}^{30} sin4x\,dx$$

Solution

$$\int_{0}^{\frac{30}{4}} sin4x\,dx = -\frac{1}{4}cos4x|_{0}^{\frac{30}{4}} = -\frac{1}{4}(cos30 - cos0)$$

$$= -\frac{1}{4}(\sqrt{\frac{3}{2}} - 1)$$

Example 6

Find the integral

$\int_{0}^{1} e^{3x}\,dx$

Solution

$$\int_{0}^{1} e^{3x}\,dx = \frac{1}{3}e^{3x}|_{0}^{1} = \frac{1}{3}(e^{3} - e^{0})$$

Example 6

$$\int_{3}^{3} (x + 9)\,dx$$

Solution

$$\int_{3}^{3} (x + 9)\,dx = 0$$

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Find the definite integral of the following functions

1- $\int_{1}^{2} 2\,dx$
2- $\int_{-2}^{0} 3x\,dx$
3- $\int_{0}^{1} x^3\,dx$
4- $\int_{3}^{3} (x+9)\,dx$
5- $\int_{1}^{3} (x^2-3)\,dx$
6- $\int_{0}^{60} cosx\,dx$
7- $\int_{-1}^{1} 2e^x\,dx$