NUSU

Faculty of Administrative Sciences

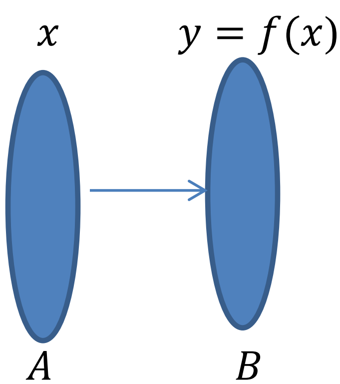
Business Mathematics-2 Semester: 2

LC(1)

Functions

Definition

A function f is a rule that assigns to each element x in a set A exactly one element, called $y = f(x)$, in a set B.



$y = f(x)$ is said to be equation

Representations of Functions

There are four possible ways to represent a function:

1. Verbally (by a description in words)

2. Numerically (by a table of values)

3. Visually (by a graph)

4. Algebraically (by an explicit formula)

The Graph of an Equation

In 1637 the French mathematician René Descartes revolutionized the study of mathematics by joining its two major fields—algebra and geometry. With Descartes's coordinate plane, geometric concepts could be formulated analytically and algebraic concepts could be viewed graphically. The power of this approach was such that within a century of its introduction, much of calculus had been developed. The same approach can be followed in your study of calculus. That is, by viewing calculus from multiple perspectives—graphically, analytically, and numerically— you will increase your understanding of core concepts. Consider the equation $3x + y = y$ The point $(2,1)$ is a solution point of the equation because the equation is satisfied (is true) when 2 is substituted for and 1 is substituted for This equation has many other solutions, such as $(1,4)$ and $(0,7)$ To find other solutions systematically, solve the original equation for y

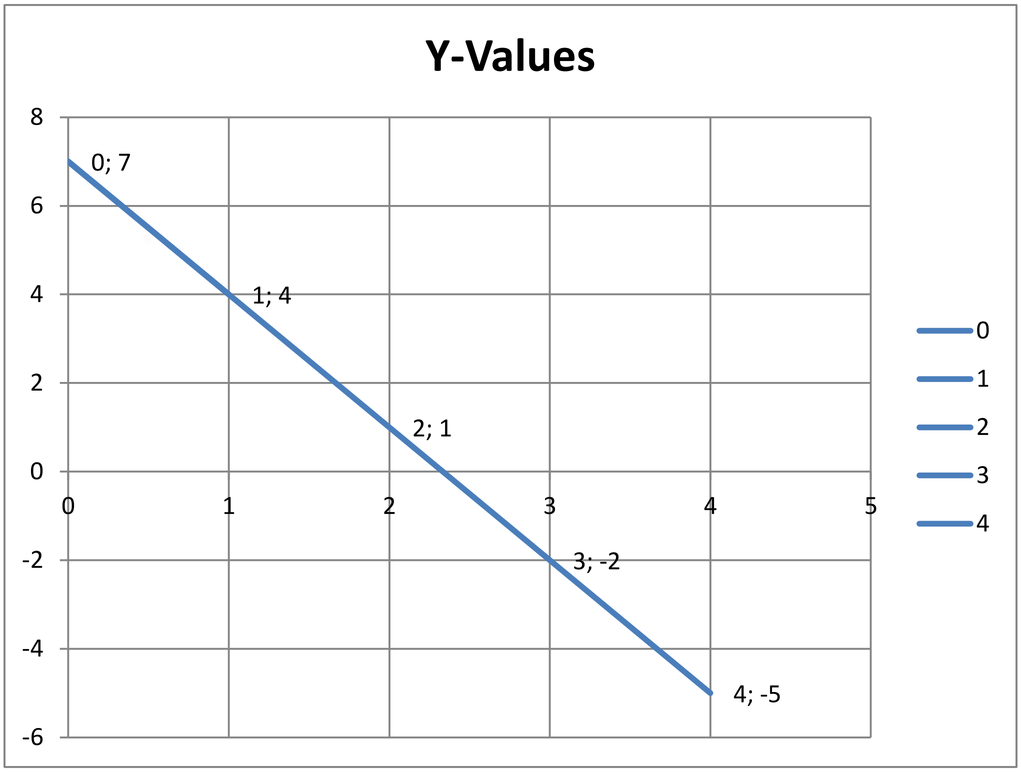
$y = 7 - 3x$ Analytic approach

Then construct a table of values by substituting several values of

x	0	1	2	3	4
y	7	4	1	-2	-5

Numerical approach

From the table ,you can see that $(0,7), (1,4), (2,1), (3,-2)$ and $(4,-5)$ are solutions of the original equation $3x + y = 7$ Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph of the equation**, as shown in Figure



$$3x + y = 7$$

EXAMPLE 1

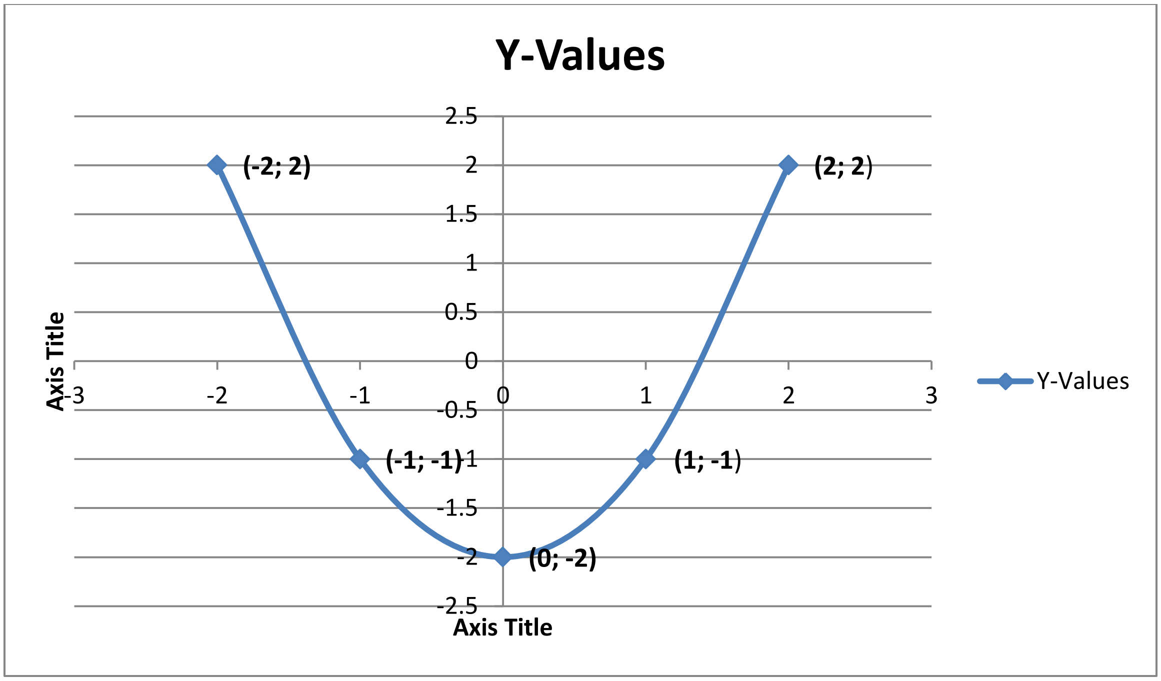
Sketching a Graph by Point Plotting

Sketch the graph of $y = x^2 - 2$

Solution First construct a table of values. Then plot the points shown in the table.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7

Finally, connect the points with a smooth curve, as shown in Figure.



This graph is a parabola.

Example

Evaluate $f(-1), f(0)$, and $f(2)$, of the following functions:

1- $f(x) = x^2 + 2x + 3$
2- $f(x) = \frac{1}{x-1}$
3- $f(x) = \sqrt{x+1}$

Solution

1- $f(x) = x^2 + 2x + 3, f(-1) = (-1)^2 + 2 \times (-1) + 3 = 2$,
$f(0) = 0^2 + 2 \times 0 + 3 = 3,\ and\ f(2) = 2^2 + 2 \times 2 + 3 = 11$

2- $f(x) = \frac{1}{x-1}$, $f(-1) = \frac{1}{-1-1} = -\frac{1}{2}$, $f(0) = \frac{1}{0-1} = \frac{1}{-1} = 1$,
$and\ f(2) = \frac{1}{2-1} = \frac{1}{1} = 1$

3- $f(x) = \sqrt{x+1}$, $f(-1) = \sqrt{-1+1} = \sqrt{0} = 0$,

$f(0) = \sqrt{0+1} = \sqrt{1} = \pm 1, and\ f(2) = \sqrt{2+1} = \sqrt{3} = 1.7321$

The piecewise function

Example

Given the function f defined by:

$$f(x) = \begin{cases} 1-x & if\ x \le 1 \\ x^2 & if\ x > 1 \end{cases}$$

Evaluate $f(0), f(1)$, and $f(2)$.

Solution

Since $0 \le 1$, we have $f(0) = 1 - 0 = 1$.

Since $1 \le 1$, we have $f(1) = 1 - 1 = 0$.

Since $2 > 1$, we have $f(2) = 2^2 = 4$.

The absolute value function

Recall that the absolute value of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so we have

$$|a| \ge 0, \qquad \text{for very number } a$$

For example

|−3| = 3, |3| = 3, and |0| = 0. In general we have

$$|a| = a \ , \qquad if\ a \ge 0$$

$$|a| = -a \ , \qquad if\ a < 0$$

(Remember that if a is negative, then $-a$ is positive.)

Then the absolute value function $f(x) = |x|$ can be written as

$$|x| = \begin{cases} x & if\ \ x \ge 0 \\ -x & if\ x < 0 \end{cases}$$

Tut(1)

A) Sketch the graph of the equation

1) $y - x = 2$
2) $y = x - 5$
3) $x^2 - 1 = y$
4) $y + 3 = 2x$
5) $y = x$

B) Given the function f defined by:

$$f(x) = \begin{cases} 2x + 4 & if\ x \leq 1 \\ x^2 - 1 & if\ x > 1 \end{cases}$$

Evaluate $f(-1), f(0), f(1)$ and $f(3)$.