Business statistics II

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- **Nonparametric**

References :

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Lecture 1

Introduction to probability distributions

Uncertainty plays an important role in our daily lives and activities as well as in business.

In some cases, uncertainty even helps heighten our interest in these activities. For example, some sports fans refuse to watch the evening news whenever

they're going to view a taped replay of a sports event that occurred earlier in the day—the replay is more exciting if they don't know who won the game.

In a business context, investment counselors cannot be sure which of two stocks will deliver the better growth over the coming year, and engineers try to reduce the likelihood that a machine will break down.

Likewise, marketers may be uncertain on the eventual success of a new product.

In everyday conversations, we all use expressions of the kind:

“Most likely our team will win this Saturday.”

“It is unlikely that the weekend will be cold.”

“I have a 50–50 chance of getting a summer job at the camp.”

The phrases “most likely,” “probable,” “quite likely,“ and so on are used to indicate the chance that an event will occur.

Probability, as a subject, provides a means of quantifying uncertainty.

In general terms, the probability of an event is a numerical value that measures how likely it is that the event will

occur.

let's just consider probability as "the chance of something happening."

Basic Terms:

EXPERIMENT: An activity or measurement that results in an outcome.

SAMPLE SPACE :All possible outcomes of an experiment.

EVENT :One or more of the possible outcomes of an experiment; a subset of the sample space.

PROBABILITY :A number between 0 and 1 that expresses the chance that an event will occur.

EXPERIMENT: An activity or measurement that results in an outcome.

The probability of an event is viewed as a numerical measure of the chance that the event will occur.

The idea is naturally relevant to situations where the outcome of an experiment or observation exhibits variation.

Although we have already used the terms "experiment" and "event," a more specific explanation is now in order.

In the present context, the term experiment is not limited to the studies conducted in a laboratory. Rather, it is used in a broad sense to include any operation of data collection or observation where the outcomes are subject to variation., a number of customers for an opinion survey, and quality inspection

of items from a production line are just a few examples.

SAMPLE SPACE :All possible outcomes of an experiment.

The sample space associated with an experiment is the collection of all possible distinct outcomes of the experiment.

Each outcome is called an elementary outcome, a simple event, or an element of the sample space

Outcome: is the result of a single trial of a probability experiment.

Example 1 :

1/ Record the number of defective light bulbs in a box of ten light bulbs. A suitable sample space is

$S = \{0,1,2,3,4,5,6,7,8,9,10\}$

2/In rolling a die, the elementary outcomes are the points 1, 2, 3, 4, 5, and 6, which together constitute the sample space.

3/ The outcome of a football game would be either a win, loss, or tie for the home team.

Each time the experiment is performed, one and only one elementary outcome can occur.

A sample space can be specified by either listing all the elementary outcomes, using convenient symbols to identify them, or making a descriptive statement that characterizes the entire collection.

we denote: **The sample space by S**

The elementary outcomes by: $e_1, e_2, e_3, \ldots$

EVENT :One or more of the possible outcomes of an experiment; a subset of the sample space.

Every outcome is an event, known as an <u>elementary event</u>.

The term is used more widely to mean any collection of outcome; an event that consists of more than one outcome is called a <u>compound event.</u>

<u>Mathematically, an event is any happening that may be represented by a subset of sample space.</u>

Example2:

The elementary event "one light bulb is defective" is represented by the subset A = {1}

The compound event "at least one light bulb is defective" is represented by the subset

B ={1,2,3,4,5,6,7,8,9,10}.

PROBABILITY :A number between 0 and 1 that expresses the chance that an event will occur.

Probability are measured on a scale from 0 to 1 ; something that is almost certain to happen has a probability close to 1, while an event that is extremely unlikely has a probability close to 0.

For example: the probability that a coin lands with heads upper most is often assumed to be 1/2.

Example 3 : Events for Coin Tossing

Toss a coin twice and record the outcome head (H) or tail (T) for each toss.

Let *A* denote the event of getting exactly one head and *B* the event of getting no heads at all.

List the sample space and give the compositions of *A* and *B*.

SOLUTION :

For two tosses of a coin, The sample space can then be listed as S = {HH, HT, TH, TT}. With the designation given above, we can also write

$$S = \{ e_1, e_2, e_3, e_4 \}$$

Consider the event A of getting exactly <u>one head</u>. Scanning the above list, we see that only the elements HT (e_2) and TH (e_3) satisfy this requirement.

Therefore, the event A has the composition

$$A = \{ e_2 , e_3 \}$$

which is, of course, a subset of S.

The event B of getting <u>no heads</u> at all consists of the single element e_4 so $B = \{e_4\}$. That is, B is a simple event as well as an event. The term "event" is a general term that includes simple events.

The probability of an event is a numerical value that represents the proportion of times the event is expected to occur when the experiment is repeated many times under identical conditions.

The probability of event A is denoted by $P(A)$.

Since a proportion must lie between 0 and 1, the probability of an event is

a number between 0 and 1. To explore a few other important properties of

probability, let us refer to the experiment in Example 1 of tossing a coin twice.

The event *A* of getting exactly one head consists of the elementary outcomes

HT(e2) and TH(e3) .

Consequently, *A* occurs if either of these occur.

Because

Proportion of times A occurs=

Proportion of times HT occurs	+	Proportion of times TH occurs

the number that we assign as $P(A)$ must be the sum of the two numbers

$P(\text{HT})$ and $P(\text{TH})$.

Guided by this example, we state some general properties of probability

The probability of an event is the sum of the probabilities assigned to all

the elementary outcomes contained in the event.

Next, since the sample space *S* includes all conceivable outcomes, in every trial

of the experiment some element of *S* must occur. Viewed as an event, *S* is certain

to occur, and therefore its probability is 1.

The sum of the probabilities of all the elements of *S* must be 1.

In summary

Probability must satisfy:

1. $0 \le P(A) \le 1$ for all events A

2. $$P(A) = \sum_{\text{all } e \text{ in } A} P(e)$$

3. $$P(S) = \sum_{\text{all } e \text{ in } s} P(e) = 1$$

We have deduced these basic properties of probability by reasoning from the definition that the probability of an event is the proportion of times the event is expected to occur in many repeated trials of the experiment.

LECTURE 2

METHODS OF ASSIGNING PROBABILITY

EQUALLY LIKELY ELEMENTARY OUTCOMES— THE UNIFORM PROBABILITY MODEL

Often, the description of an experiment ensures that each elementary outcome is as likely to occur as any other. For example, consider the experiment of rolling a fair die and recording the top face. The sample space can be listed as

$$S = \{e1, e2, e3, e4, e5, e6\}$$

Where e_1 stands for the elementary outcome of getting the face 1, and similarly, $e_2, \ldots, e_6$.

Without actually rolling a die, we can deduce the probabilities.

Because a fair die is a symmetric cube, each of its six faces is as likely to appear as any other.

In other words, each face is expected to occur one-sixth of the time. The probability assignments should therefore be

$$P(e1) = P(e2) = \ldots\ldots\ldots = P(e6) = 1/6$$

and any other assignment would contradict the statement that the die is fair.

We say that rolling a fair die conforms to a uniform probability model because the total probability 1 is evenly apportioned to all the elementary outcomes.

What is the probability of getting a number higher than 4? Letting *A* denote

this event, we have the composition

$$A = \{e_5 , e_6 \}\text{,so}$$

$$P(A) = P(e_5) + P(e_6) = 1/6+1/6=2/6=1/3$$

When the elementary outcomes are modeled as equally likely, we have a uniform probability model. If there are k elementary outcomes in S, each is assigned the probability of $1/k$.
An event A consisting of m elementary outcomes is then assigned

$$P(A) = m/k = \frac{\text{No. of elementary outcomes in } A}{\text{No. of elementary outcomes in } S}$$

Example 1 *The Uniform Probability Model for Tossing a Fair Coin:*

Find the probability of getting exactly one head in two tosses of a fair coin.

SOLUTION: there are four elementary outcomes in the sample space:

***S*= {HH, HT, TH, TT}.** evry concept of a fair coin implies that the four elementary outcomes in *S* are equally likely We therefore assign the probability1/4 to each of them. The event *A* = [One head] has two elementary outcomes—namely, HT and TH. Hence,

$$P (A) = 2/4= ½ = 0.5$$

Example 2 :*Random Selection and the Uniform Probability Model*

Suppose that among 50 students in a class, 42 are right-handed and 8 lefthanded. If one student is randomly selected from the class, what is the probability that the selected student is left-handed?

SOLUTION: The intuitive notion of random selection is that each student is as likely to be selected as any other. If we view the selection of each individual student as an elementary outcome, the sample space consists of 50 *e*'s of which 8 are in the event "left-handed."

Consequently,

P[Left-handed]= **8/50=0.16**

Note: **Considering that the selected student will be either left-handed (*L*) or right-handed (*R*), we can write the sample space as**

$$S = \{L, R\},$$

but we should be aware that the two elements *L* and *R* are not equally Likely.

PROBABILITY AS THE LONG-RUN RELATIVE FREQUENCY

When the assumption of equally likely elementary outcomes is not tenable,

how do we assess the probability of an event? The only recourse is to repeat the experiment many times and observe the proportion of times the

event occurs. Letting *N* denote the number of repetitions (or trials) of an experiment,

we set

$$\text{Relative frequency of event } A \text{ in } N \text{ trials} = \frac{\text{No. of times } A \text{ occurs in } N \text{ trials}}{N}$$

For instance, let *A* be the event of getting a 6 when rolling a die. If the die is rolled 100 times and 6 comes up 23 times, the observed relative frequency of *A* would be $= \frac{23}{100} = 0.23$

In the next 100 tosses, 6 may come up 18 times. Collecting these two sets together, we have N=200 trials with the observed relative frequency

$$\frac{23+18}{200} = \frac{41}{200} = 0.205$$

Probability as Long-Run Relative Frequency

We define $P(A)$, the probability of an event A, as the value to which the relative frequency stabilizes with increasing number of trials. Although we will never know $P(A)$ exactly, it can be estimated accurately by repeating the experiment many times.

EVENT RELATIONS

the three most basic event relations:

complement, union, and intersection.

These event relations will then motivate some laws of probability.

The event operations are conveniently described in graphical terms.We first

represent the sample space as a collection of points in a diagram, each identified

with a specific elementary outcome.

Venn diagram:

Example 3: Venn Diagram for Coin Tossing

Make a Venn diagram for the experiment of tossing a coin twice and indicate the following events.

A: Tail at the second toss

B: At least one head

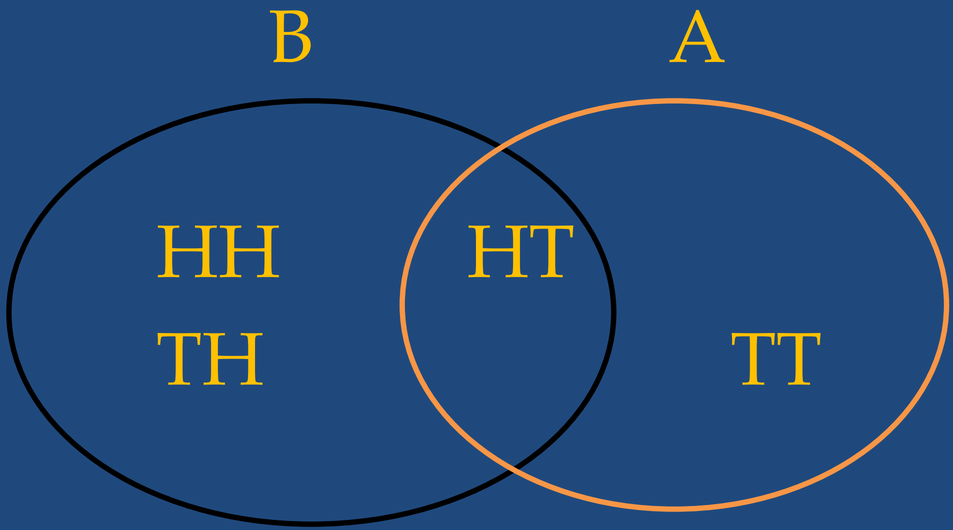
SOLUTION :Here the sample space is

S = {HH, HT, TH, TT}, and the two events have

the compositions A = {HT, TT}, B = {HH, HT, TH}.

Figure shows the

Venn diagram.



We now proceed to define the three basic event operations and introduce the corresponding symbols. The **complement** of an event *A*, denoted by $\bar{A}$ is the set of all elementary outcomes that are not in *A*. The occurrence of $\bar{A}$ means that ***A does not occur.***

Rule for Complementary Events

$$P(\bar{A}) = 1 - P(A) \quad \text{or} \quad P(A) = 1 - P(\bar{A}) \quad \text{or} \quad P(A) + P(\bar{A}) = 1$$

Example : **If the probability that a person lives in an industrialized country of the world is 1/5, find the probability that a person does not live in an industrialized country.**

Solution

***P*(not living in an industrialized country($\bar{A}$) =**

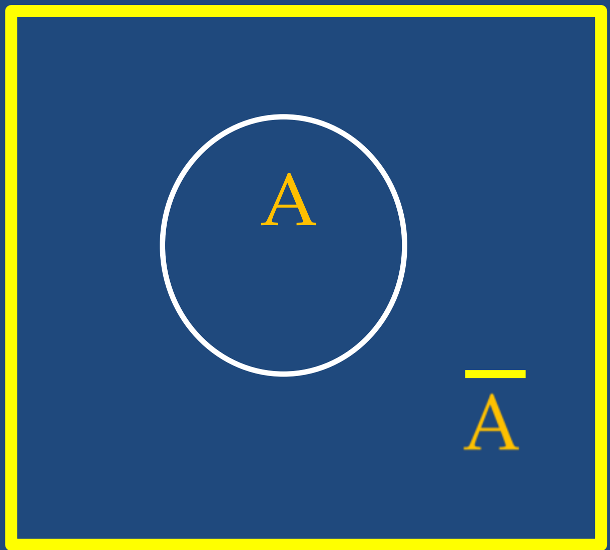
1- *P*(living in an industrialized country(A)

1- 1/5 = 4/5

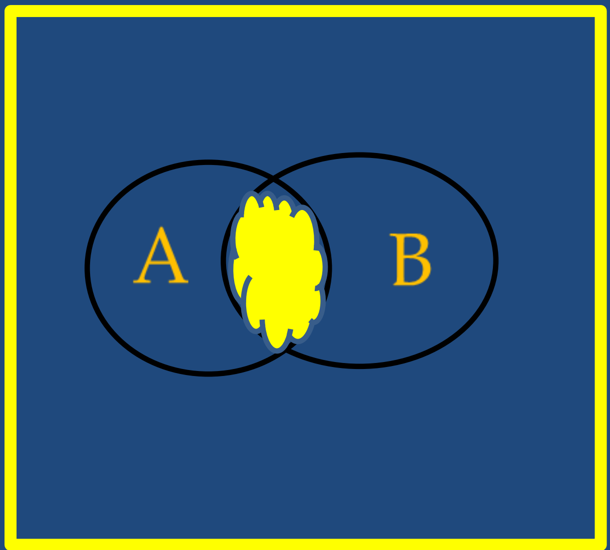
The **union** of two events A and B, denoted by $A \cup B$, is the set of all elementary outcomes that are in A, B, or both.

The occurrence of $A \cup B$ means that *either A or B or both occur*.

The **intersection** of two events A and B, denoted by $A \cap B$, is the set of all elementary outcomes that are in A and B. The occurrence of $A \cap B$ means that *both A and B occur*.

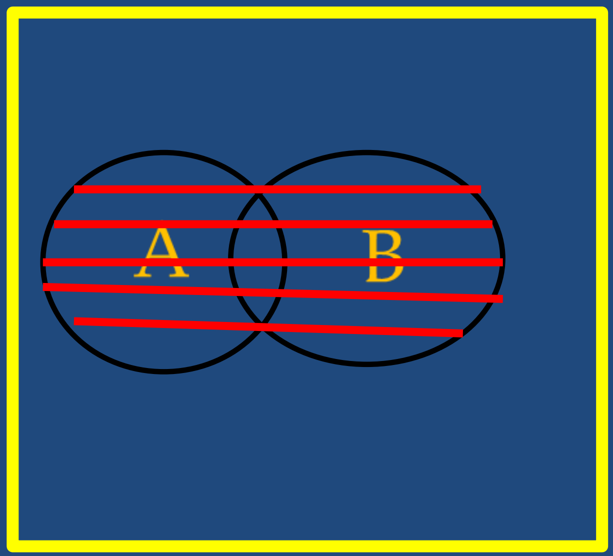


Complement

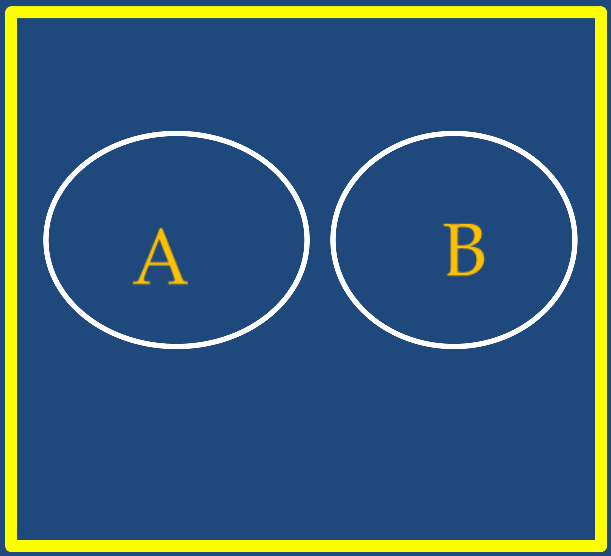


Intersection

 $= A \cap B$



Union = $A \cup B$



Incompatible events

Note that $A \cup B$ is a larger set containing A as well as B, where as $A \cap B$ is the common part of the sets A and B. Also it is evident from the definitions that

$A \cup B$ and $B \cup A$ represent the same event,

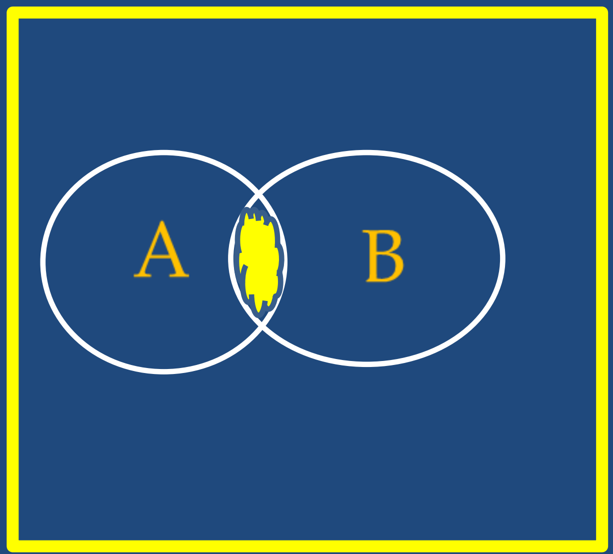
while $A \cap B$ and $B \cap A$ are both expressions for the intersection of A and B.

The operations of union and intersection can be extended to more than two events.

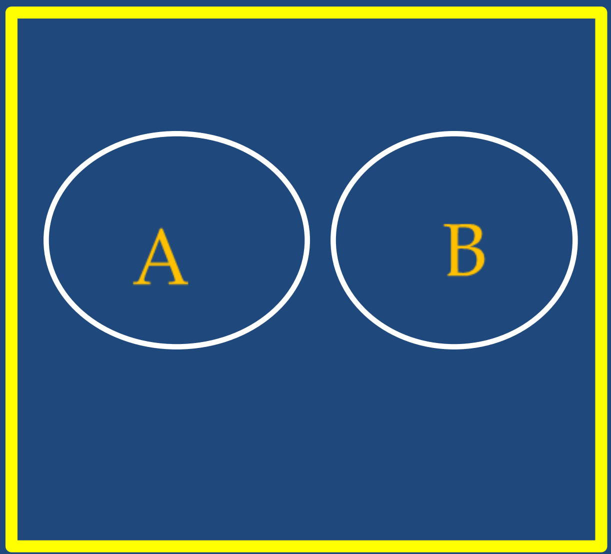
For instance, $A \cup B \cup C$ stands for the set of all points that are in *at least one* of A, B, and C, where as ABC represents the *simultaneous occurrence* of all three events.

Two events A and B are called **incompatible** or **mutually exclusive** if their intersection $A \cap B$ is empty. Because incompatible events have no elementary outcomes in common.

Two events are mutually exclusive if they cannot occur at the same time (i.e., they have no outcomes in common).



not mutually exclusive



mutually exclusive

Addition Rule 1

The probability of two or more events can be determined by the *addition rules*. The first addition rule is used when the events are mutually exclusive.

When two events *A* and *B* are mutually exclusive, the probability that *A* or *B* will occur is

$$P(A \mathbf{U} B) = P(A) + P(B)$$

Example: A city has 9 coffee shops: 3 Starbuck's, 2 Caribou Coffees, and 4 Crazy Mocho Coffees.

If a person selects one shop at random to buy a cup of coffee, find the probability that it

is either a Starbuck's or Crazy Mocho Coffees

Solution

Since there are 3 Starbuck's and 4 Crazy Mochos, and a total of 9 coffee shops,

P(Starbuck's or Crazy Mocho) =

P(Starbuck's) + P(Crazy Mocho)=

3/9 +4/9 =7/9

Addition Rule 2

If A and B are *not* mutually exclusive, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example:

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females.

If a staff person is selected, find the probability that the subject is a nurse or a male.

Staff	Females	Males	Total
Nurses	**7**	**1**	**8**
Physicians	3	2	5
total	10	3	13

P(nurse **U** male) = P(nurse) +P(male) - P(male ∩ nurse)

8/13 +3/13 – 1/13 = 10/13

In summary, then, when the two events are mutually exclusive, use addition rule 1.

When the events are not mutually exclusive, use addition rule 2.

The probability rules can be extended to three or more events. For three mutually exclusive events *A*, *B*, and *C*,

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

For three events that are *not* mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Example: Determine whether these events are mutually exclusive.

a. Roll a die: Get an even number, and get a number less than 3.

No

b. Roll a die: Get a prime number (2, 3, 5), and get an odd number.

No

c. Roll a die: Get a number greater than 3, and get a number less than 3.

Yes

d. Select a student in your class: The student has blond hair, and the student has blue eyes.

No

e. Select a student in your college: The student is a sophomore, and the student is a business major.

No

f. Select any course: It is a calculus course, and it is an English course.

Yes

g. Select a registered voter: The voter is a Republican, and the voter is a Democrat.

Yes